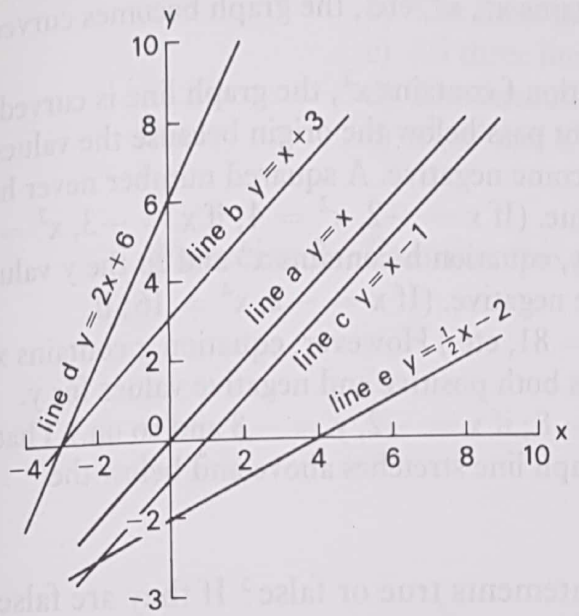
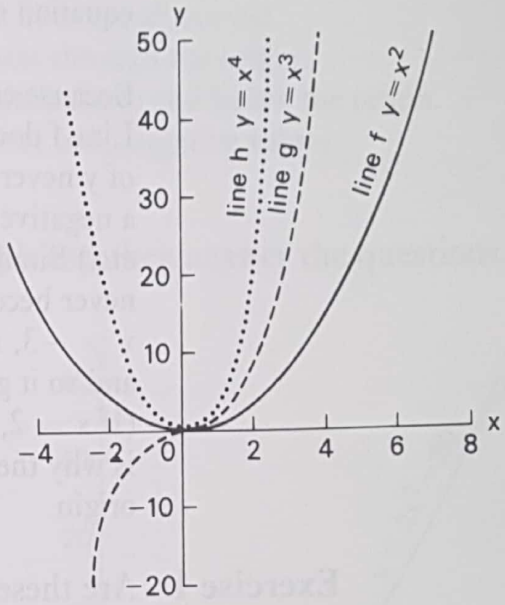
UNIT FIVE Graphs

SECTION A: MATHEMATICAL EQUATIONS





Mathematical equations contain variables. If there are only two variables, it is possible to show their relationship on a graph.

The simplest equation is y = x. The graph of y = x is a straight line. In this equation, the values of x and y always remain equal. (If x = 1, then y = 1; if x = −3, then y = −3, etc.) This graph line (line a) passes through the *origin* of the graph.

Sometimes the equation of a straight line contains a *constant*. For example, in the equation y = x+3, x and y are the two *variables* and 3 is the constant. If an equation contains a constant then the graph line does not pass through the origin. Look at lines b and c. If the constant is positive, then the line cuts the y *axis* above the origin. If the constant is negative, then the line cuts the y axis below the origin.

Lines a, b and c all have the same *gradient*. That is to say they all slope at the same angle to the x axis. Now look at lines d and e. Line d has a higher gradient and line e has

a lower gradient. This is because the *coefficients* of x are different in these equations. In equations a, b and c, x has a coefficient of 1. That is why the graph lines all have the same gradient. However, in equation d the coefficient of x is 2 and in equation e the coefficient is $\frac{1}{2}$. If the coefficient of x is high, the gradient is high. Similarly, if the coefficient is low, the gradient is low.

These five lines are all straight. However, if a graph equation contains x^2, x^3, etc., the graph becomes curved.

Because equation f contains x^2, the graph line is curved. Line f does not pass below the origin because the values of y never become negative. A squared number never has a negative value. (If $x = -2$, $x^2 = 4$; if $x = -3$, $x^2 = 9$, etc.) Similarly, equation h contains x^4 and so the y values never become negative. (If $x = -2$, $x^4 = 16$; if $x = -3$, $x^4 = 81$, etc.) However, equation g contains x^3, and so it gives both positive and negative values for y. (If $x = 2$, $y = 8$; if $x = -2$, $y = -8$ and so on.) That is why the graph line stretches above and below the origin.

Exercise 1 Are these statements true or false? If they are false, write corrected statements.

1. $y = x$ (line a)

a) If $x = 10$, $y = 10$.
b) The graph line passes through the origin.
c) The equation contains a constant.

2. $y = x+3$ (line b)

a) If $x = 4$, then $y = 7$.
b) The graph line passes through the origin.
c) The equation contains a negative constant.
d) Lines a and b have the same gradient.

3. $y = x-1$ (line c)

a) if $x = -2$, $y = 0$.
b) The graph line cuts the y axis below the origin.
c) The equation contains a negative constant.
d) The coefficient of x is 2.

4. $y = 2x+6$ (line d)

a) If $x = 0$, $y = 6$.
b) The graph line has the highest gradient.
c) The equation contains the constant 2.

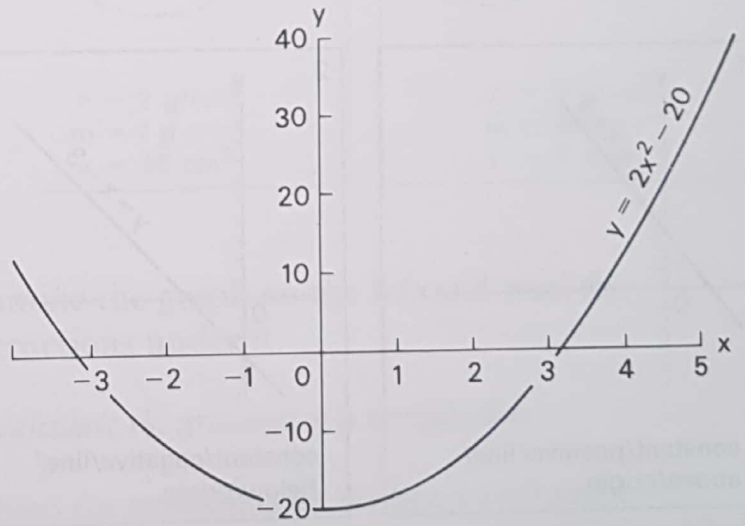
5. $y = \frac{1}{2}x - 2$ (line e)

a) If $x = 4$, $y = -2$.
b) Line e has a lower gradient because the coefficient of x is low.
c) The line cuts the y axis above the origin.
d) The equation contains the constant $\frac{1}{2}$.

6. $y = x^2$, $y = x^3$, $y = x^4$ (lines f, g, h)

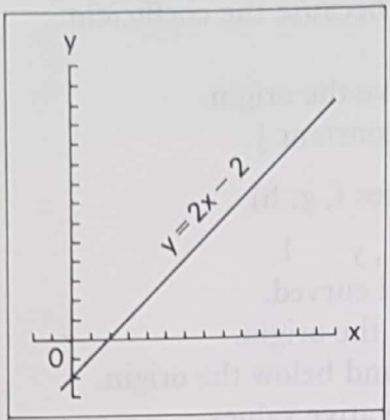
a) In all three graphs, if $x = 1$, $y = 1$.
b) The three graph lines are all curved.
c) All three lines pass through the origin.
d) The lines all stretch above and below the origin.
e) x^2, x^3 and x^4 never have negative values.

Exercise 2 Examine this graph and then answer the questions.



1. Is the graph line curved?
2. Does y have both positive and negative values?
3. Where does the graph line cut the y axis?
4. How many times does the graph line cut the x axis?
5. What is the equation?
6. If there is a constant in the equation, what is its value?
7. If x has a coefficient, what value does it have?
8. If $x = 1$ what does y equal?
   If $x = -2$ what does y equal?
   If $x = 4$ what does y equal?
   If $x = 5$ what does y equal?

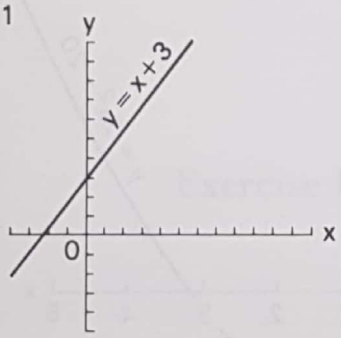
Exercise 3 Make sentences from the pictures. Look at the example first.



equation/constant/line/origin

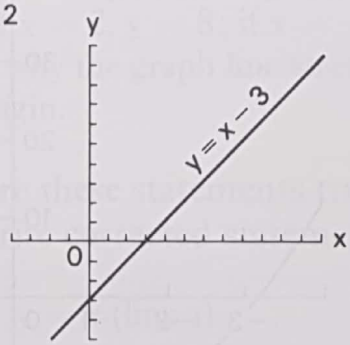
If the *equation* contains a *constant*, the *line* does not pass through the *origin*.

1



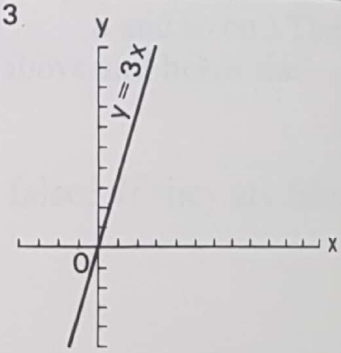
constant/positive/line/above/origin

2



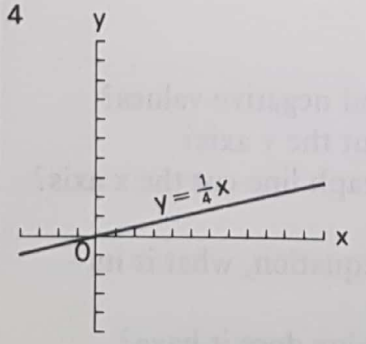
constant/negative/line/below/origin

3



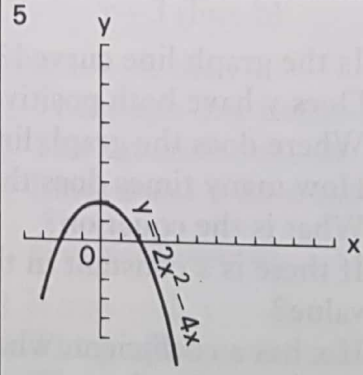
x/high coefficient/gradient/high

4



x/low coefficient/gradient/low

5



equation/x^2/graph/curved

6

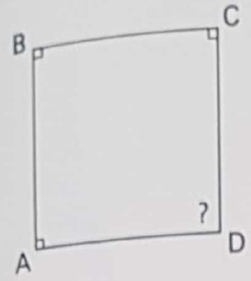
$x = -1, x^4 = 1$

$x = -2, x^4 = 16$

$x = -3, x^4 = 91$

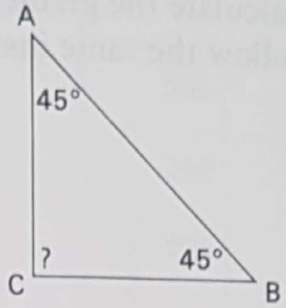
x/negative/x^4/positive

7



angles A, B, C/right angles/
angle D = ?

8

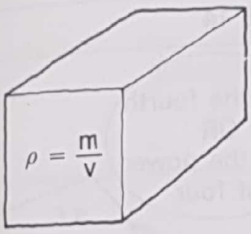


angles A, B = 45°/
angle C = ?

9

angle B = 90°, angle
C = 60°/angle A = ?

10

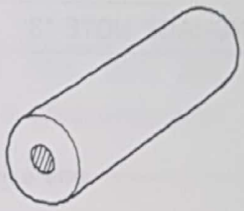


ρ = ? g/cm^3
m = 500 g
v = 100 cm^3

11

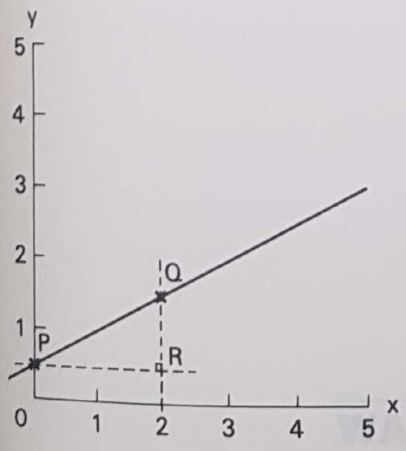
ρ = 2 g/cm^3
m = ? g
v = 16 cm^3

12



ρ = 9 g/cm^3
m = 900 g
v = ? cm^3

Exercise 4 Examine the graph on the left and read the instructions under it.



To calculate the gradient of a straight line.

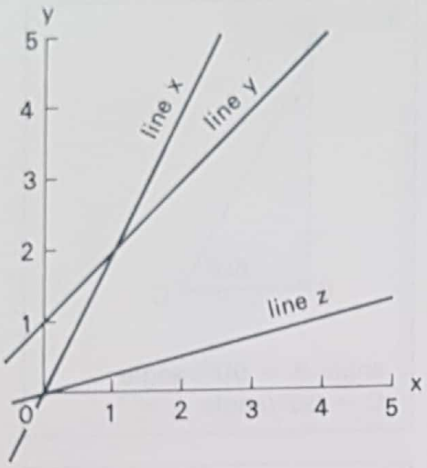
1. Find the two points on the line x = 0 and x = 2.
2. Mark the two points with your pencil and then label them P and Q.
3. Construct a right-angled triangle PQR. First draw a line parallel to the y axis through Q. Label the third point of the triangle R.
4. Measure the lines QR and PR. (QR = 1; PR = 2)
5. The gradient is calculated with the formula $\frac{QR}{PR}$.

   Therefore, the gradient is $\frac{1}{2}$.

Finally, write a short report. Start like this:

To calculate the gradient of a graph line.

1. First, the two points x = 0 and x = 2 were . . . . . .



Calculate the gradients of the three lines on the left
Follow the same instructions.

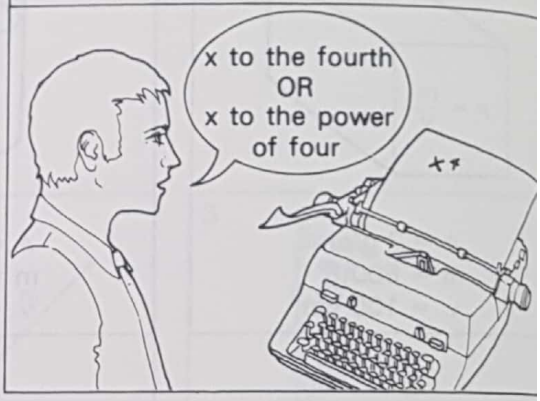
LANGUAGE NOTE 13

one axis

BUT

two axes

LANGUAGE NOTE 14



a number
a value
a variable
a constant
a coefficient
a gradient
an origin
an axis

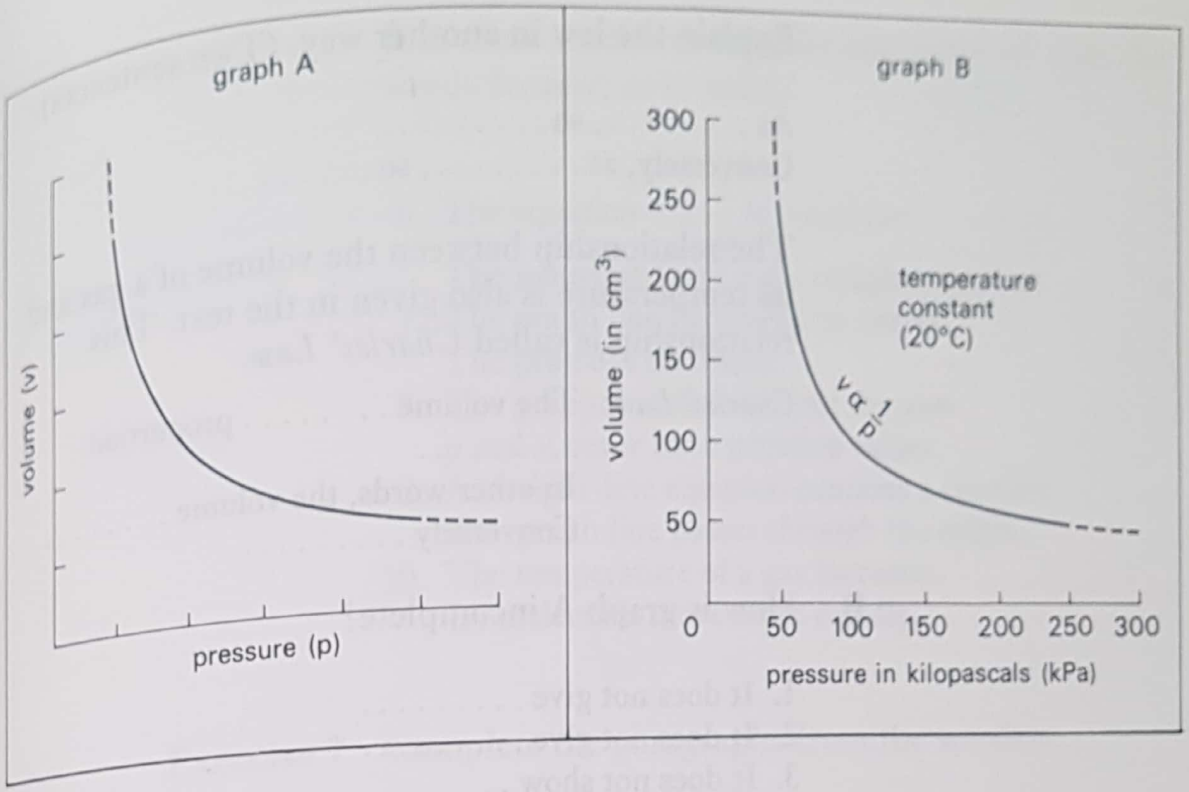
if . . . . . then . . . . . .

similarly

right-angled

SECTION B: BOYLE'S LAW

The two graphs on page 69 give the relationship between the pressure of a gas and its volume. This relationship is called Boyle's Law. As the pressure of a gas increases, so its volume decreases. Conversely, as the pressure decreases, so the volume increases. In other words, the pressure of a gas varies in inverse proportion to its volume ($v \propto \frac{1}{p}$).



Although both the graphs give this information, graph A is incomplete. Although the axes are labelled, the units of measurement are not given. Similarly, the graph equation is not given although the graph line is correctly drawn.

Moreover, although the axes are labelled correctly, the values of p and v are not shown. It is impossible to use the graph unless the values of p and v are given. Unless the values are shown, the graph provides no numerical information at all.

Furthermore, the graph is inaccurate unless it includes the words 'Temperature Constant'. The volume of a gas is affected by temperature. The volume varies in direct proportion to the temperature. In other words, the volume of a gas decreases as its temperature decreases. Conversely, as the temperature increases, the volume also increases. Therefore, the equation $v \propto \frac{1}{p}$ is inaccurate unless the temperature remains constant, and the graph shown is correct only when $T = 20°C$.

Exercise 5 Study the text and then complete the sentences.

A. Describe *Boyle's Law* in one sentence.
Boyle's Law: The pressure of . . . . . proportion . . . . . .

Explain the law in another way. (Two sentences)

As . . . . . , so . . . . . .
Conversely, as . . . . . , so . . . . . .

The relationship between the volume of a gas and its temperature is also given in the text. This relationship is called *Charles' Law*.

Charles' Law: The volume . . . . . proportion . . . . . .
In other words, the volume . . . . . .
Conversely . . . . . .

B. How is graph A incomplete?

1. It does not give . . . . . .
2. It does not give . . . . . .
3. It does not show . . . . . .
4. It does not include . . . . . .

Complete sentences 5, 6, 7 and 8 with the information from sentences 1, 2, 3 and 4. Use the word *unless*.

5. It is impossible to use the graph unless . . . . . given.
6. The graph is incomplete . . . . . .
7. The graph provides no numerical information . . . . . .
8. The graph is inaccurate . . . . . .

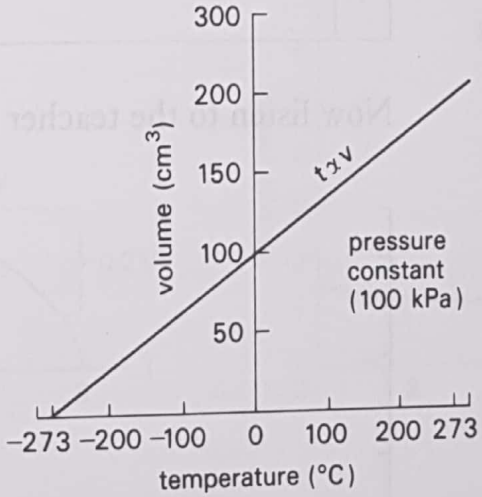
Exercise 6 Join these sentences to make one sentence. Use the word *although*.

1. Both graphs show Boyle's Law.
   One graph is incomplete.
2. Graph A gives some information.
   It is incomplete.
3. The axes are labelled.
   The units of measurement are not given.
4. The graph equation is not given.
   The graph line is correctly drawn.
5. The values of p and v are not shown.
   The axes are correctly labelled.

Join these sentences to make one sentence. Use the words *because*, *as* or *unless*.

6. The equation $v \propto \frac{1}{p}$ is inaccurate.
   The temperature of a gas remains constant.
7. The graph line becomes less curved.
   The pressure increases.
8. The graph line does not cut the axes.
   p and v never have negative values.
9. A straight-line equation contains a constant.
   The graph line passes through the origin.
10. The temperature of a gas increases.
    Its molecules move more rapidly.

Exercise 7 Complete the paragraphs from the wordlist.



above	*decreases*	*Law*
Although	*direct*	*marked*
Conversely	*impossible*	*negative*
	increases	*relationship*
		to
		words

The graph shows the . . . . between the temperature of a gas and its volume. This relationship is called Charles' . . . . . The temperature of a gas is in . . . . proportion to its volume. In other . . . . , as the temperature . . . . , so the volume also increases. . . . . , as the temperature decreases, the volume of the gas . . . . .

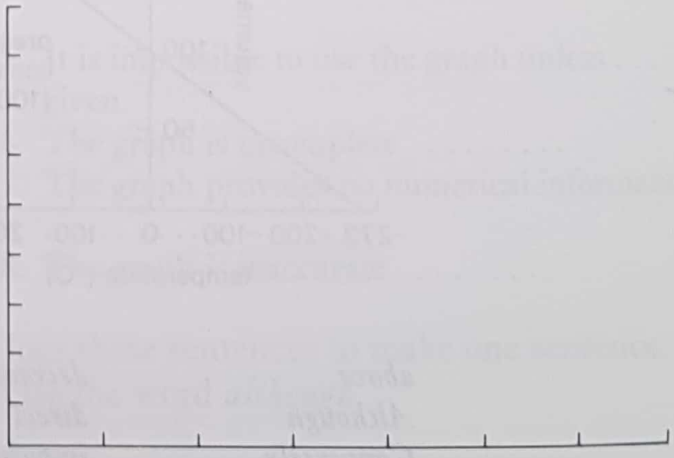
The vertical axis has no . . . . values. . . . . it is possible to

have negative values for temperature, it is . . . . to have a negative value for volume. The temperature values are . . . . along the horizontal axis from −273°C . . . . +273°C. At higher temperatures (i.e. . . . . 273°C) Charles' Law is no longer very accurate.

Exercise 8 Fill in the table.

	days of the week	abbreviations
1.		
2.		
3.		
4.		
5.		
6.		
7.		

Now listen to the teacher and follow his instructions

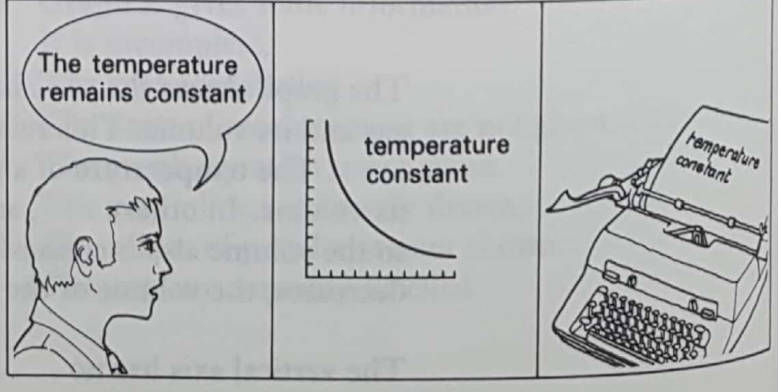


LANGUAGE NOTE 15

$v \propto t$: in direct proportion to

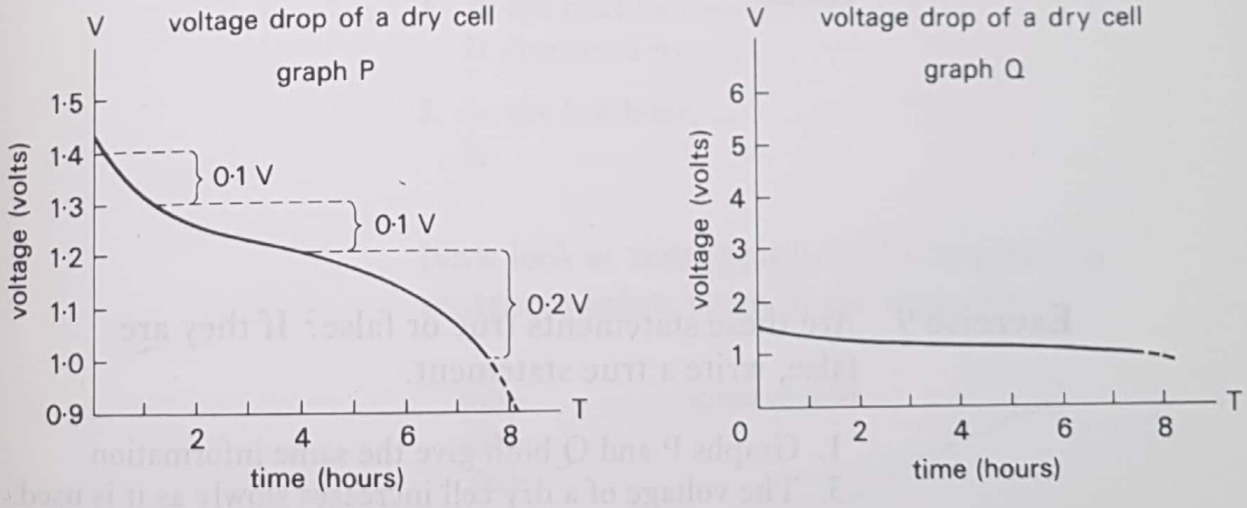
$v \propto \frac{1}{p}$: in inverse proportion to

LANGUAGE NOTE 16



a relationship	moreover	numerical
a law	furthermore	(in)complete
a week		(in)accurate
	in direct/inverse proportion	
as . . . . . (so)	to	conversely
unless		
although	in other words	include

SECTION C: TIME GRAPHS



Graphs P and Q both show the voltage drop in the same dry cell. The cell was in continuous use for eight hours. When the cell was new, the voltage was approximately 1.4 V. As the cell was used, the voltage decreased. When the voltage decreased to less than 1.0 V, the current became very weak. After eight hours the voltage was no longer strong enough to provide much power.

It is important to use suitable scales in a graph. In graph Q, the voltage scale is unsuitable. The scale shows values from 0 V to 5 V. It is only necessary to show values from 0.9 V to 1.5 V. Moreover, the voltage scale is too small. It is difficult to take any accurate measurements from graph Q. In graph P, only the necessary voltage values are given. It is much easier to measure the voltage drop when the scale is larger.

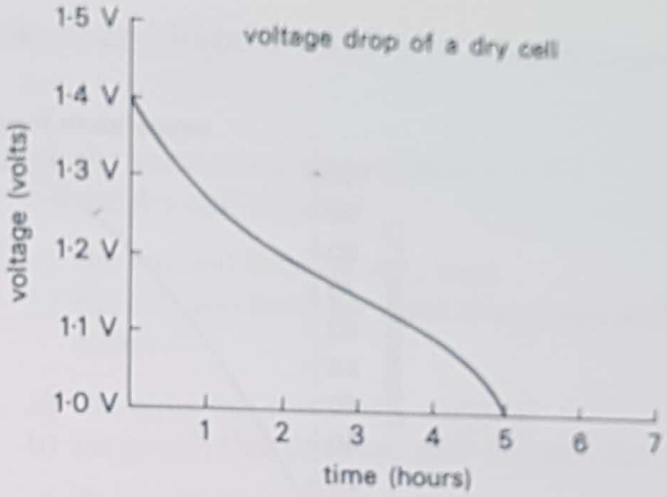
The voltage decreased quite rapidly in the first hour. It decreased from approximately 1.4 V to 1.3 V. In other words, it decreased by 0.1 V in one hour. In the next three hours, the decrease was much slower. It decreased from 1.3 V to 1.2 V. In other words, it only decreased by 0.1 V in three hours. In the next two and a half hours it decreased more rapidly again. It decreased by 0.2 V in two and a half hours. The dry cell became useless when it was used continuously for more than eight hours.

The graphs do not include one necessary piece of information. Although they give measurements of the voltage drop, they do not give any information about the use of the cell. We do not know if the cell was used in a torch or in a radio, for example. A torch bulb uses much more current than a radio and so needs more power. Unless we know the use of the cell, the graph is not very useful.

Exercise 9 Are these statements true or false? If they are false, write a true statement.

1. Graphs P and Q both give the same information.
3. The voltage of a dry cell increases slowly as it is used.
3. After continuous use for eight hours, the cell still provided a little power.
4. Graph P has a more suitable voltage scale than graph Q.
5. In graph Q, the voltage scale is too small.
6. In graph P, the voltage scale is too large.
7. It is easy to take voltage measurements from graph P.
8. It is easy to take accurate measurements from graph Q.
9. The graphs include information about the use of the cell.
10. This cell was used in a radio.

Exercise 10 This cell was used in a large torch. Examine the graph and the example under it.

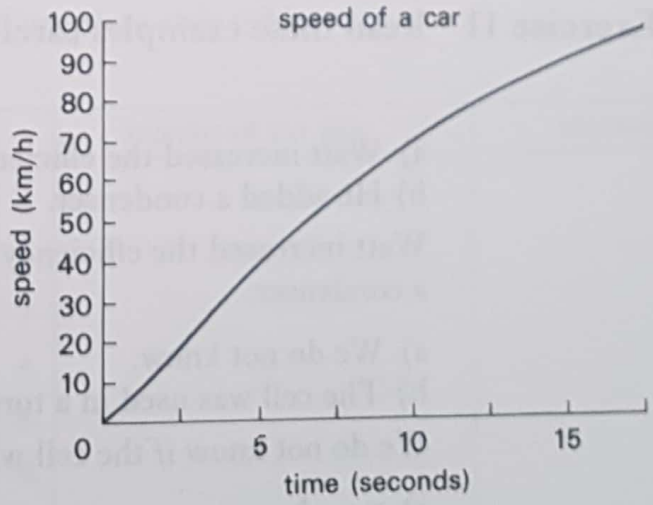


In the first two hours, the voltage decreased from 1.4 V to 1.2 V. It decreased by 0.2 V in two hours.

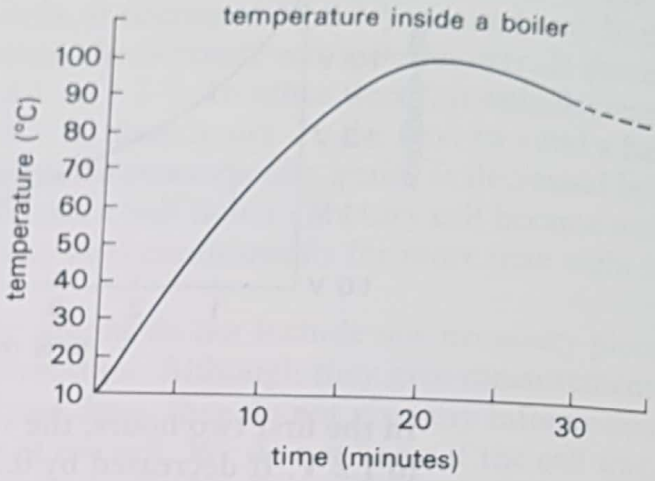
Complete these sentences in the same way.

1. In the next two hours, ...
   It decreased by . . . . . .
2. In the last hour, ...
   It . . . . . .

Now look at these graphs and complete the sentences below them in the same way.



3. In the first five seconds, ...
   It . . . . . .
4. In the next . . . . . .
   It . . . . . .
5. In the last . . . . . .
   It . . . . . .



6. In the first . . . . . .
   It . . . . . .
7. In . . . . . .
   It . . . . . .
8. In . . . . . .
   It . . . . . .

Exercise 11 Read these examples carefully.

a) Watt increased the efficiency of his engine.
b) He added a condenser.

Watt increased the efficiency of his engine *when* he added a condenser.

a) We do not know.
b) The cell was used in a torch.

We do not know *if* the cell was used in a torch.

a) $x = 4$
b) $x^2 = 16$

If $x = 4$, $x^2 = 16$
OR
When $x = 4$, $x^2 = 16$
OR
$x = 4$ if (or when) $x^2 = 16$

Join a) and b) below with *if* or *when* or both.

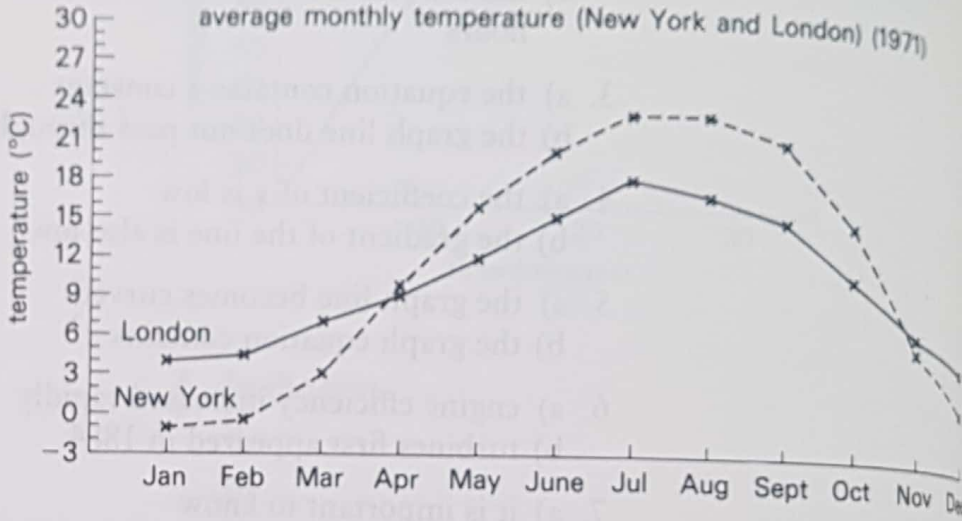
1. a) the voltage was approximately 1.4 V
   b) the dry cell was new
2. a) the current became very weak
   b) the cell was used for more than seven and a half hours
3. a) the equation contains a constant
   b) the graph line does not pass through the origin
4. a) the coefficient of x is low
   b) the gradient of the line is also low
5. a) the graph line becomes curved
   b) the graph equation contains x^2
6. a) engine efficiency increased rapidly
   b) turbines first appeared in 1884
7. a) it is important to know
   b) the carbon content of steel is high or low
8. a) the equation $v \propto \frac{1}{p}$ is inaccurate
   b) the temperature is allowed to vary

Exercise 12 Complete this table.

	months of the year	abbreviations
1.		
2.		
3.		
4.		
5.		
6.		
7.		
8.		
9.		
10.		
11.		
12.		

Below there is a discontinuous time graph. It gives information about the average monthly temperature in New York and in London.

Study the graph and listen to your teacher.



How long did the temperature stay above / below . . . . °C?

How far did the temperature rise / fall between . . . . and . . . .?

What was the temperature difference between London and New York in . . . .?

LANGUAGE NOTE 17

in 1884		on Monday
in July	BUT	on July 21st
in the first hour		on the third day

a drop
a second
a month
a difference

(an) acceleration

rise
stay

(dis)continuous
(un)suitable
(un)necessary

average

useful
useless

continuously

any

how long . . . .?
how far . . . .?