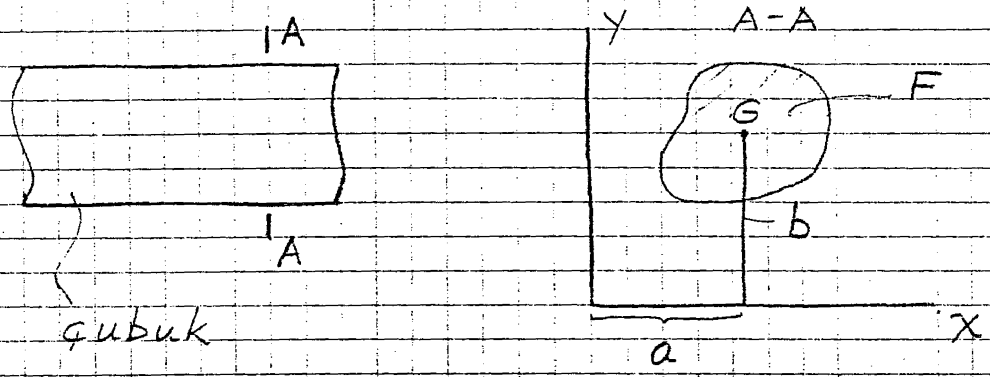
BÖLÜM 1

ATALET MOMENTLERİ



$F \rightarrow$ Kesit Alan

$G \rightarrow$ "F" nin ağırlık merkezi

$G \rightarrow ?$

$\downarrow$

$(a, b) \rightarrow ?$

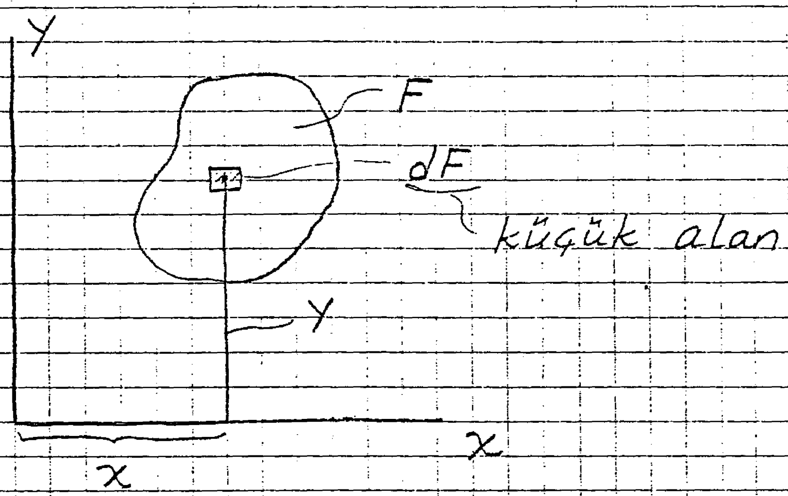
a: x-koor.

b: y koor.

$$\left.\begin{aligned} b &= \frac{S_x}{F} \\ a &= \frac{S_y}{F} \end{aligned}\right\} \quad (*)$$

$S_x \rightarrow$ F'nin x-etrafındaki statik momenti

$S_y \rightarrow$ F'nin y-etrafındaki statik momenti



$$S_x = \int_F y \, dF$$

$$S_y = \int_F x \, dF$$

$$[S_x], [S_y] \rightarrow L^3 \rightarrow cm^3, m^3 \text{ vs.}$$

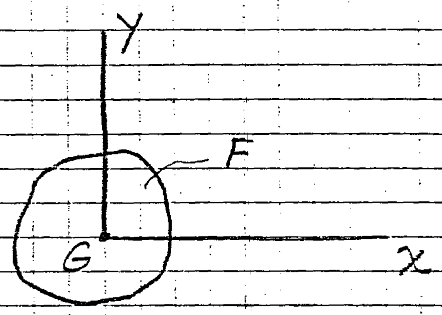
(*) :

$$\Rightarrow \quad S_x = b * F = \int_F y \, dF$$

$$S_y = a * F = \int_F x \, dF$$

ÖZEL HAL :

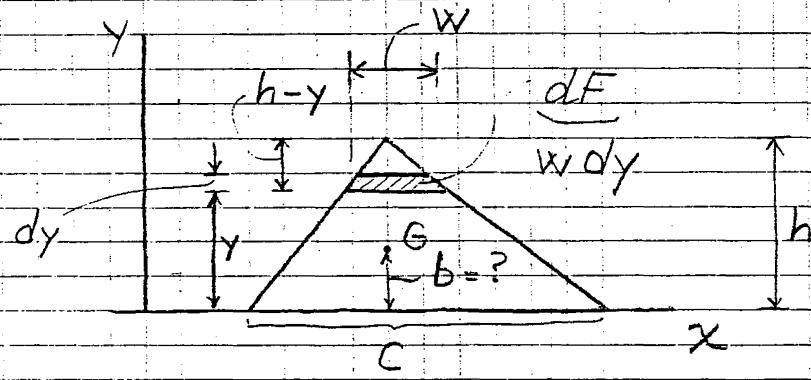
(xy) nin orijini G ile çakışıyor :



$$\Rightarrow \quad a = b = 0 \Rightarrow \quad S_x = S_y = 0$$

ÖRNEK:

Üçgen kesit ağırlık merkezi ?



$$b = \frac{S_x}{\underbrace{F}_{\frac{ch}{2}}}$$

$$S_x = \int_F y\, \underbrace{dF}_{w\,dy}$$

$$\frac{h-y}{h} = \frac{w}{c} \Rightarrow w = \frac{c}{h}(h-y)$$

$$\Rightarrow S_x = \int_0^h y * \underbrace{\overbrace{\frac{c}{h}(h-y)}^{w}\, dy}_{dF}$$

$$\Rightarrow S_x = \frac{c}{h}\int_0^h (hy - y^2)\, dy$$

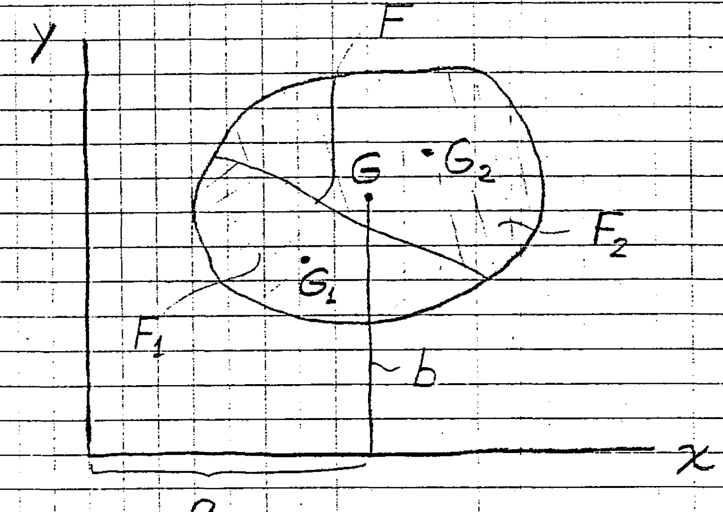
$$= \frac{c}{h}\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h$$

$$= \frac{c}{\cancel{h}}\underbrace{\left[\frac{h^3}{2} - \frac{h^3}{3}\right]}_{\frac{h^{\cancel{3}\,2}}{6}} - 0$$

$$\Rightarrow S_x = \frac{ch^2}{6}$$

$$\Rightarrow b = \frac{ch^2}{6} * \frac{2}{ch}$$

$$\Rightarrow \boxed{b = \frac{h}{3}}$$



$G \rightarrow F$'nin ağırlık merkezi

$\downarrow$

(a, b)

$G_1 \rightarrow F_1$'in ağırlık merkezi

$\downarrow$

(a_1, b_1)

$G_2 \rightarrow F_2$'nin ağırlık merkezi

$\downarrow$

(a_2, b_2)

toplam kesit alanı

$$F = F_1 + F_2$$

$$\left.\begin{array}{l} G_1 \rightarrow (a_1, b_1) \\ G_2 \rightarrow (a_2, b_2) \end{array}\right\} \text{biliniyor}$$

$$G \rightarrow (a, b) \quad ?$$

$$b = \frac{\overbrace{S_x}^{b_1 F_1 + b_2 F_2}}{\underbrace{F}_{F_1 + F_2}}$$

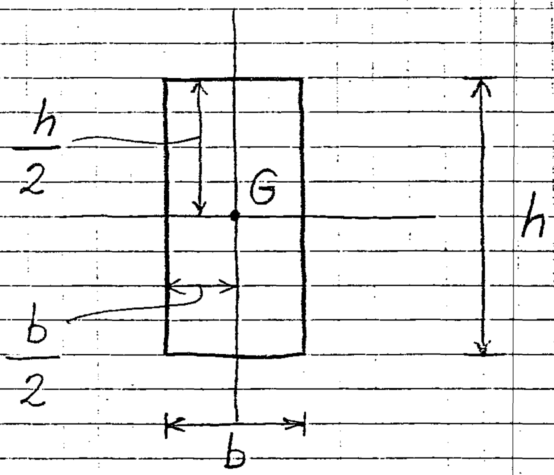
$$\Rightarrow \boxed{b = \frac{\sum bF}{\sum F}}$$

$$a = \frac{\overbrace{S_y}^{a_1 F_1 + a_2 F_2}}{\underbrace{F}_{F_1 + F_2}}$$

$$\Rightarrow \boxed{a = \frac{\sum aF}{\sum F}}$$

Genel Kural:

Eğer kesitin bir simetri ekseni varsa ağırlık merkezi bu eksen üzerinde olur.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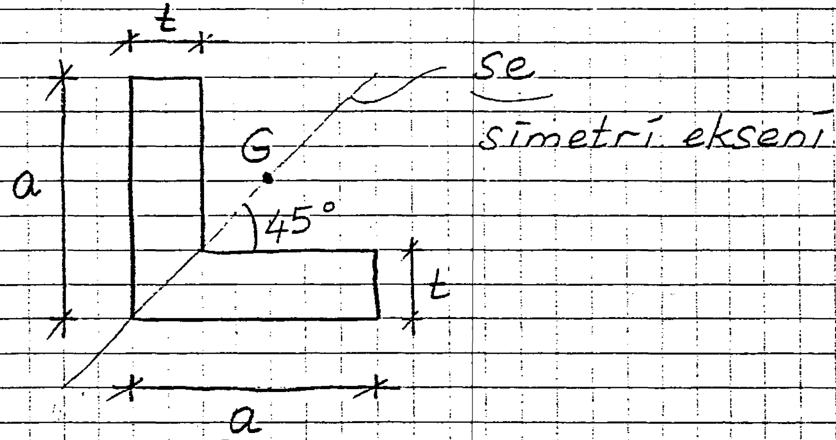
b/2
b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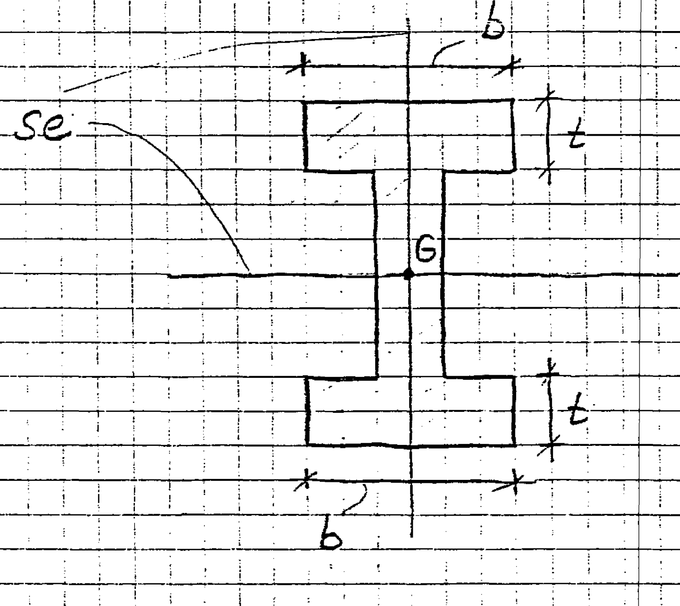
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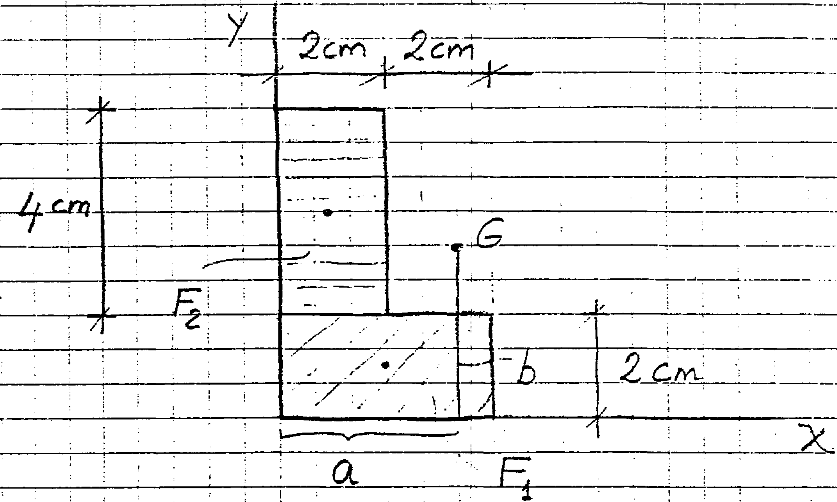
ÖRNEK:



$G \rightarrow (a, b)$?

$$\left.\begin{array}{l} F_1 = 8\ cm^2 \\ F_2 = 8\ cm^2 \end{array}\right\} \Rightarrow \sum F = 16\ cm^2$$

$$b = \frac{S_x}{\underbrace{F}_{16}} \qquad \underbrace{8 * 1 + 8 * 4}_{40}$$

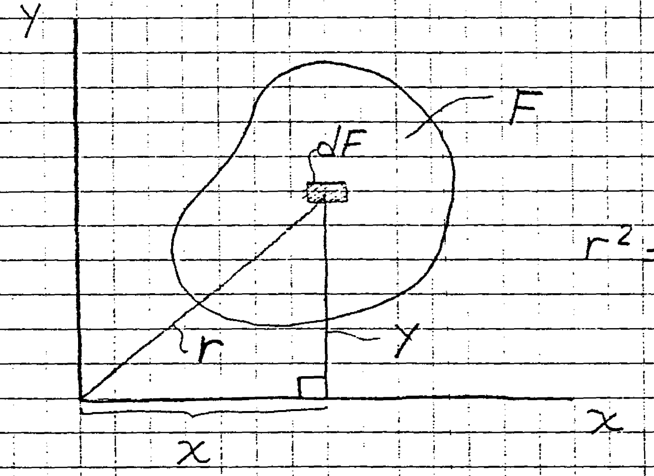
$$\Rightarrow \boxed{b = 2.5\ cm}$$

$$a = \frac{S_y}{\underbrace{F}_{16}} \qquad \underbrace{8 * 2 + 8 * 1}_{24}$$

$\left.\begin{array}{l} a \\ b \end{array}\right\}$ +− olabilir

$$\Rightarrow \boxed{a = 1.5\ cm}$$

ATALET MOMENTLERİ



$$r^2 = x^2 + y^2$$

$$S_x = \int_F y \, dF$$

$$I_x = \int_F y^2 \, dF \rightarrow$$ F nin x-etrafındaki atalet momenti

$$I_y = \int_F x^2 \, dF \rightarrow$$ F nin y-etrafındaki atalet momenti

$$I_{xy} = \int_F xy \, dF \rightarrow$$ F nin çarpım atalet momenti

$$I_o = \int_F \underbrace{r^2}_{x^2+y^2} dF \rightarrow$$ F nin kutupsal atalet momenti

$$\Rightarrow \quad I_o = \underbrace{\int_F x^2\, dF}_{I_y} + \underbrace{\int_F y^2\, dF}_{I_x}$$

$$\Rightarrow \quad \boxed{I_o = I_x + I_y}$$

$$I_x, I_y, I_o \rightarrow (+)$$

$$I_{xy} \rightarrow \gtrless 0$$

$$[I_x], [I_y], [I_{xy}], [I_o] \rightarrow L^4 \rightarrow cm^4$$

m^4

vs.

$$i_x = \sqrt{\frac{I_x}{F}} \rightarrow I_x \text{ için atalet yarıçapı}$$

$I_x > 0$

$$i_y = \sqrt{\frac{I_y}{F}} \rightarrow I_y \text{ için atalet yarıçapı}$$

$I_y > 0$

I_{xy} için atalet yarıçapı yok!

$$i_o = \sqrt{\frac{I_o}{F}} \rightarrow I_o \text{ için atalet yarıçapı}$$

$I_o = I_x + I_y$

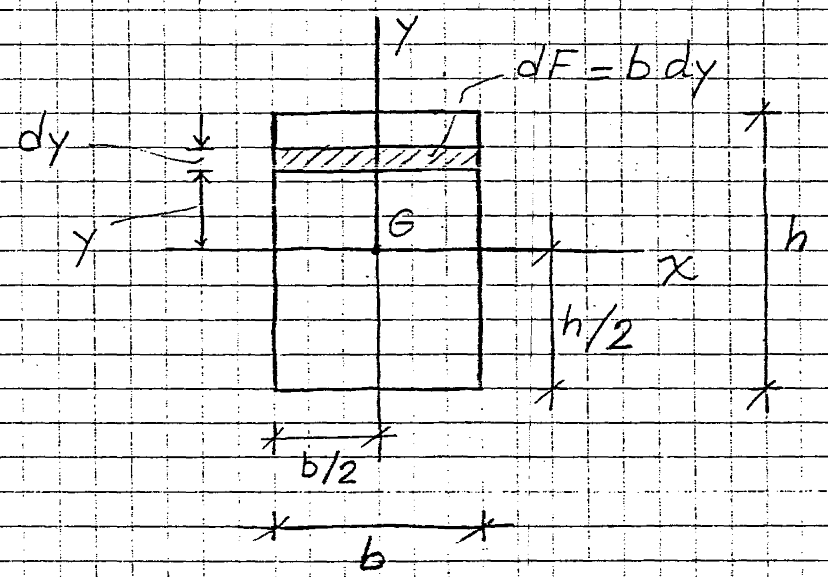
$$\Rightarrow \quad i_o = \sqrt{\underbrace{\frac{I_x}{F}}_{i_x^2} + \underbrace{\frac{I_y}{F}}_{i_y^2}}$$

$$\Rightarrow \quad i_o = \sqrt{i_x^2 + i_y^2}$$

$$[i_x],\ [i_y],\ [i_o] \rightarrow L \rightarrow cm, m, vs.$$

ÖRNEK:

Dikdörtgen kesit için atalet momenti?



$$I_x = \int_F y^2 \overbrace{dF}^{b\,dy} = \int_{-h/2}^{h/2} y^2\, b\, dy$$

$$\Rightarrow \quad I_x = b\int_{-h/2}^{h/2} y^2\,dy = b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$= \frac{b}{3}\left[\underbrace{\left(\frac{h}{2}\right)^3}_{\frac{h^3}{8}} - \underbrace{\left(-\frac{h}{2}\right)^3}_{-\frac{h^3}{8}}\right]$$

$$\underbrace{\frac{h^3}{8} - \left(-\frac{h^3}{8}\right)}_{\frac{h^3}{4}}$$

$$\Rightarrow \quad \boxed{I_x = \frac{bh^3}{12}} \quad \rightarrow [I_x] \rightarrow L^4$$

$$\Rightarrow \quad \boxed{I_y = \frac{hb^3}{12}}$$

$$\Rightarrow \quad I_o = I_x + I_y$$

$$= \frac{bh^3}{12} + \frac{hb^3}{12}$$

$$\boxed{I_{xy} = \int xy\,dF = 0}$$

$$i_x = \sqrt{\frac{I_x}{F}} = \frac{h}{\sqrt{12}}$$

$\frac{bh^3}{12}$

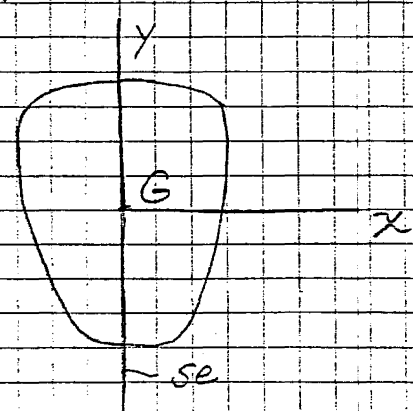
$\cancel{bh}$

$$i_y = \sqrt{\frac{I_y}{F}} = \frac{b}{\sqrt{12}}$$

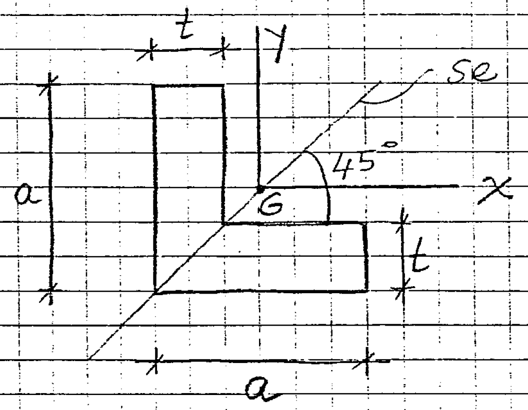
$$i_o = \sqrt{i_x^2 + i_y^2} = \sqrt{\frac{h^2}{12} + \frac{b^2}{12}}$$

Genel Kural:

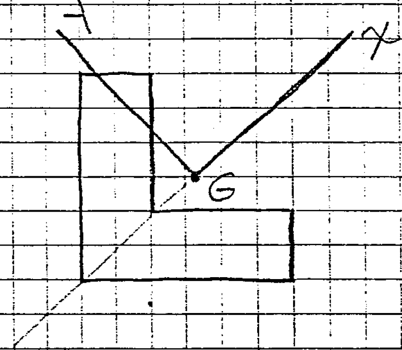
Eğer kesit bir simetri eksenine sahip ve xy takımının eksenlerinden birisi bu simetri ekseni ile çakışacak şekilde seçilirse ve buna ek olarak xy takımının orijini G ağırlık merkezinde ise kesitin çarpım atalet momenti I_{xy} sıfır olur.



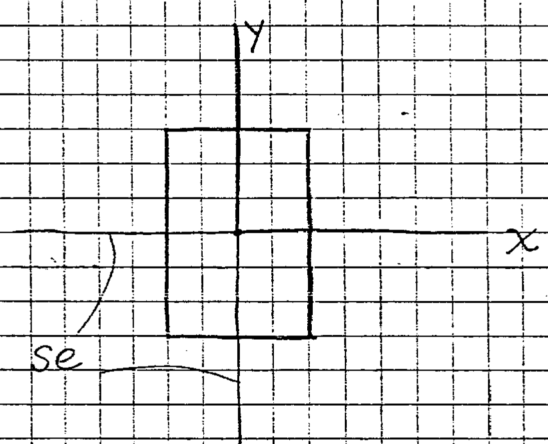
$I_{xy} = 0$



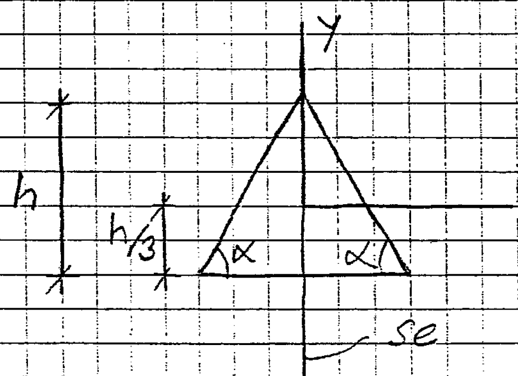
$\Rightarrow \quad I_{xy} \neq 0$



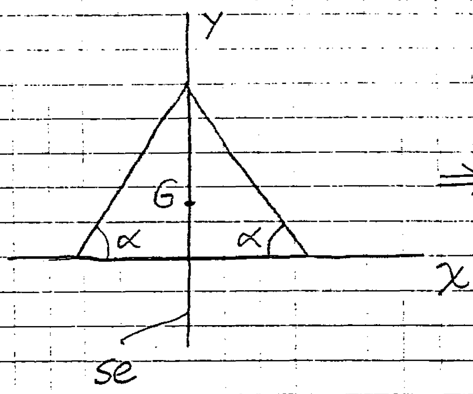
$\Rightarrow \quad I_{xy} = 0$



$\Rightarrow \quad I_{xy} = 0$



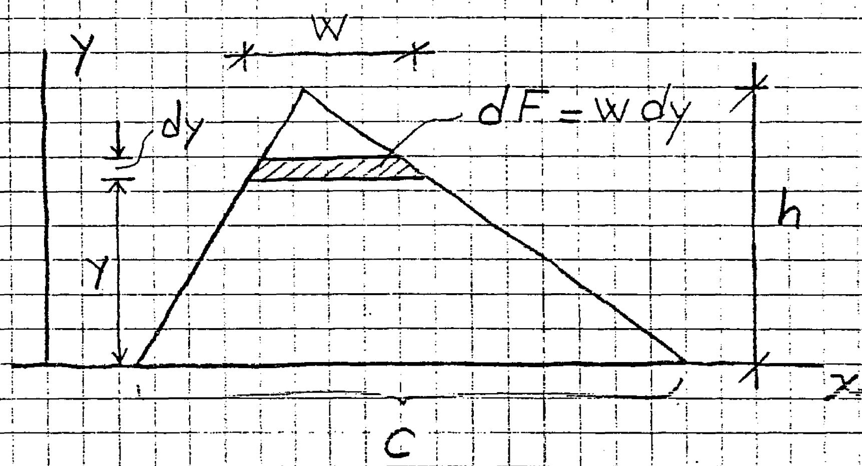
$\Rightarrow \quad I_{xy} = 0$



$$\Rightarrow I_{xy} \neq 0$$

ÖRNEK:

Üçgen kesitin tabanına göre atalet momenti



$I_x = ?$

$$I_x = \int_F y^2 \, dF \quad w\,dy$$

$$\frac{h-y}{h} = \frac{w}{c} \Rightarrow w = \frac{c}{h}(h-y)$$

$$\Rightarrow \quad I_x = \int y^2 \overbrace{\underbrace{\frac{c}{h}(h-y)}_{w}\,dy}^{dF}$$

$$= \frac{c}{h}\int_0^h \left(hy^2 - y^3\right)\,dy$$

$$= \frac{c}{h}\left[\frac{hy^3}{3} - \frac{y^4}{4}\right]_0^h$$

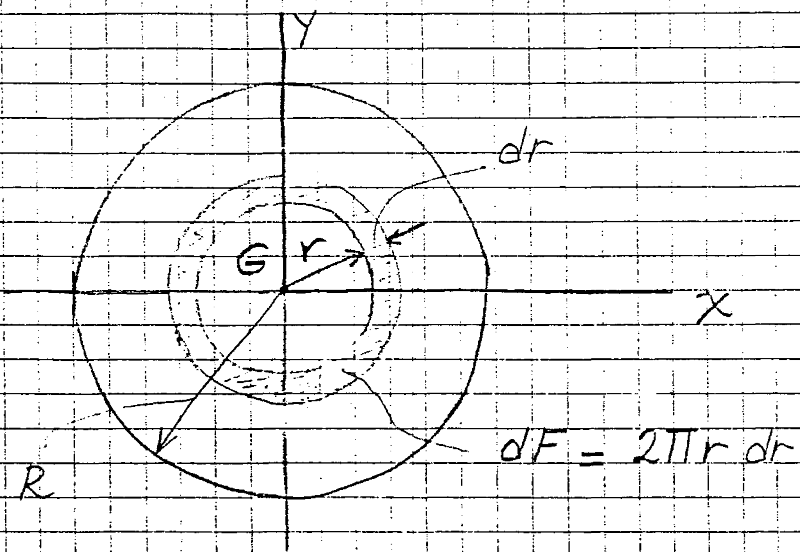
$$= \frac{c}{h}\underbrace{\left[\frac{h^4}{3} - \frac{h^4}{4}\right]}_{\frac{h^{\cancel{4}\,3}}{12}} - 0$$

$$\Rightarrow \quad \boxed{I_x = \frac{ch^3}{12}}$$

ÖRNEK:

Dairesel kesit için atalet momentleri

$$\left.\begin{matrix} I_x \\ I_y \\ \underbrace{I_{xy}}_{=0} \end{matrix}\right\} ?$$



$$I_x = I_y$$

$$I_o = I_x + \underbrace{I_y}_{=I_x}$$

$$\Rightarrow \boxed{I_x = \frac{I_o}{2} = I_y} \quad (*)$$

$$I_o = \int_F r^2 \underbrace{dF}_{2\pi r\, dr}$$

$$= 2\pi \int_0^R r^3 dr$$

$$\Rightarrow \quad I_o = 2\pi \left.\frac{r^4}{4}\right|_0^R$$

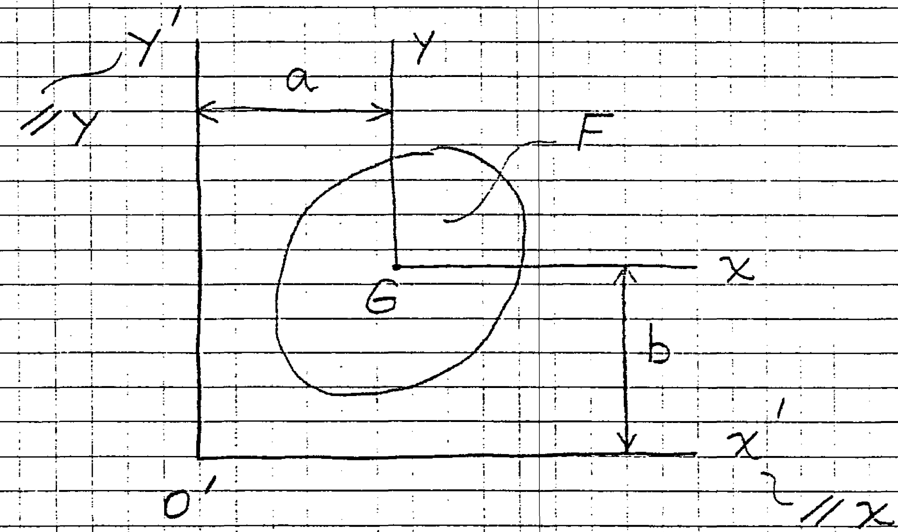
$$= \cancel{2}\pi \frac{R^4}{\cancel{4}_2} \Rightarrow \boxed{I_o = \frac{\pi R^4}{2}}$$

*:

$$\Rightarrow \quad \boxed{I_x = I_y = \frac{\pi R^4}{4}}$$

EKSENLERİN KAYDIRILMASI

(Paralel Olarak)



$$I_x = \int y^2 \, dF$$

$$I_y = \int x^2 \, dF$$

$$I_{xy} = \int xy \, dF$$

Hesap edilmiş ⇓ Biliniyor

$$S_x = \int y \, dF = 0$$

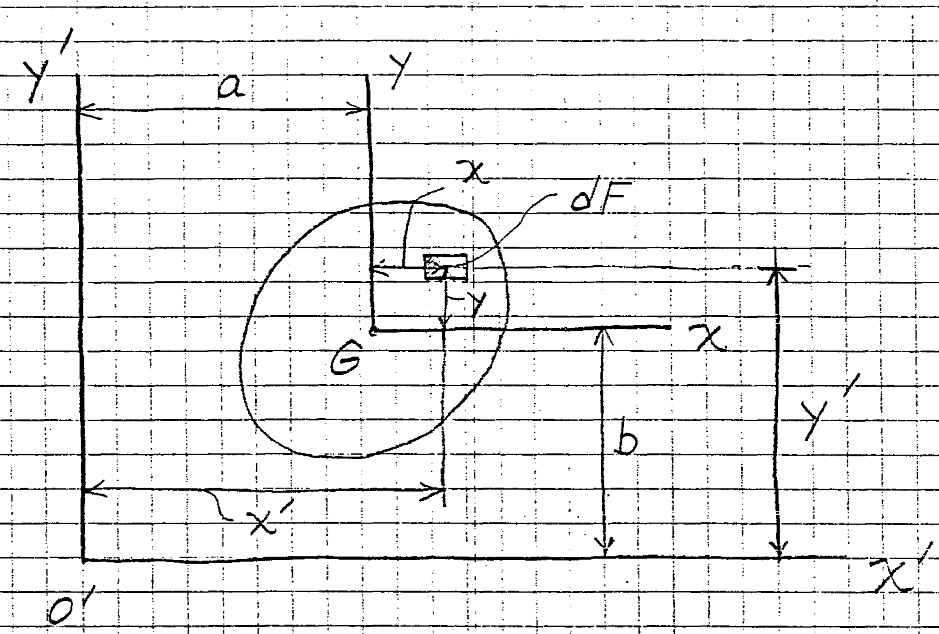
$$S_y = \int x \, dF = 0$$

$$\left.\begin{array}{l} I_{x'} \\ I_{y'} \\ I_{x'y'} \end{array}\right\} ?$$

$$I_{x'} = \int_F y'^2 \, dF$$

$$I_{y'} = \int_F x'^2 \, dF$$

$$I_{x'y'} = \int_F x'y' \, dF$$



$(a, b) \rightarrow G$'nin $(x'y')$ takımındaki koordinatları

$$y' = y + b$$

$$x' = x + a$$

$$I_{x'} = \int y'^2 \, dF \qquad (y+b)^2$$

$$= \int \underbrace{(y+b)^2}_{y^2 + 2by + b^2} dF$$

$$= \underbrace{\int y^2 \, dF}_{I_x} + 2b \overbrace{\int y \, dF}^{0} + b^2 \underbrace{\int dF}_{F}$$

$$\Rightarrow \boxed{I_{x'} = I_x + b^2 F} \quad b^2 F > 0$$

$$I_{y'} = \int x'^2 \, dF \qquad (x+a)^2$$

$$\Rightarrow \boxed{I_{y'} = I_y + a^2 F} \quad a^2 F > 0$$

$$I_{x'y'} = \int x'y'\, dF \qquad (y+b)$$

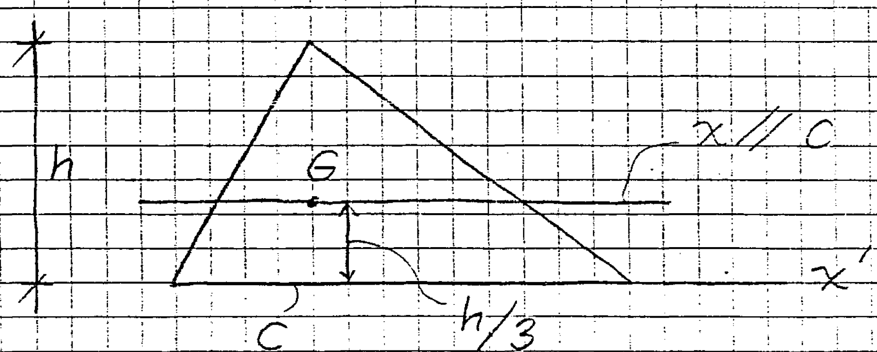
$$(x+a)$$

$$\Rightarrow \boxed{I_{x'y'} = I_{xy} + ab\,F} \quad \gtrless 0$$

$a, b \rightarrow \oplus$ veya $\ominus$ olabilir.

ÖRNEK:

Üçgen kesitin ağırlık merkezinden geçen ve tabanına paralel eksen etrafındaki atalet momenti



$I_x = ?$

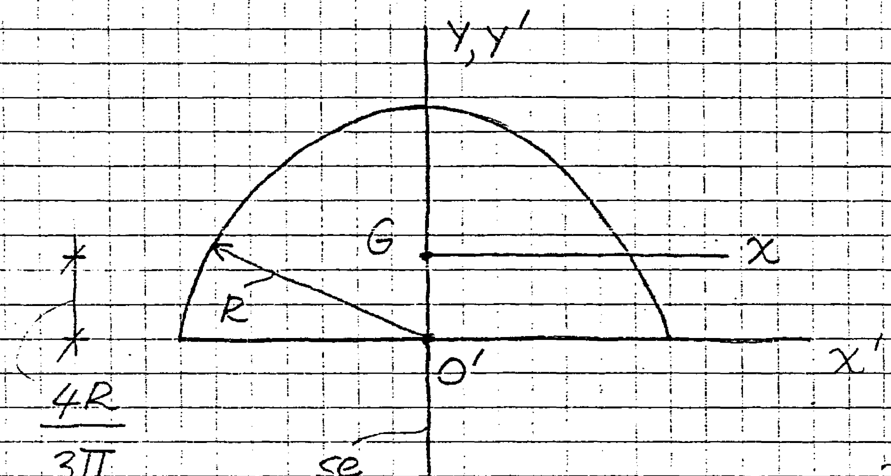
$$\underbrace{I_{x'}}_{\frac{ch^3}{12}} = I_x + \overbrace{b^2}^{(h/3)^2} \underbrace{F}_{\frac{ch}{2}} \qquad \left(b = +\frac{h}{3} \right)$$

$$\Rightarrow I_x = \frac{ch^3}{12} - \underbrace{\frac{h^2}{9} * \frac{ch}{2}}_{\frac{ch^3}{18}}$$

$$\Rightarrow \boxed{I_x = \frac{ch^3}{36}}$$

ÖRNEK:

Yarım dairesel kesit için atalet momenti



$I_x = ?$

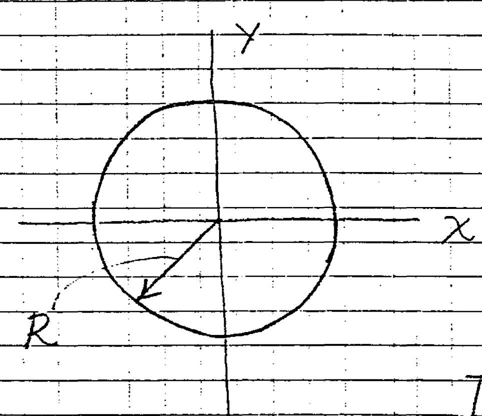
$I_y = ?$

$\underbrace{I_{xy} = ?}_{=0}$

$$\left(b = \frac{S_{x'}}{\underbrace{F}_{\frac{\pi R^2}{2}}} \right)$$

Hatırla:

tüm dairesel kesit



$$I_x = I_y = \frac{\pi R^4}{4}$$

$$\Rightarrow \quad I_{x'} = \frac{\pi R^4}{8}$$

$$I_{y'} = \frac{\pi R^4}{8}$$

$$I_y = I_{y'} \Rightarrow \boxed{I_y = \frac{\pi R^4}{8}}$$

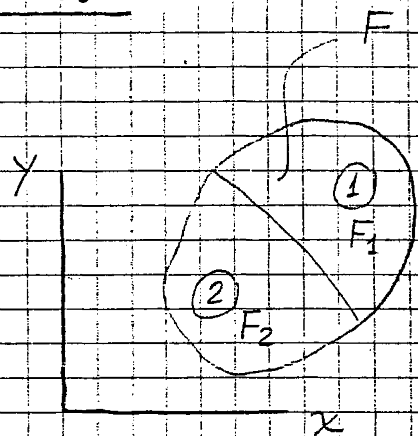
$$\underbrace{I_{x'}}_{\frac{\pi R^4}{8}} = I_x + b^2 \underbrace{F}_{\frac{\pi R^2}{2}}$$

$$\left(b = +\frac{4R}{3\pi} \right)$$

$$\Rightarrow \quad I_x = \frac{\pi R^4}{8} - \underbrace{\frac{\cancel{16}^{\,8} R^2}{9\pi^{\cancel{2}}} * \frac{\pi R^2}{\cancel{2}}}_{\frac{8R^4}{9\pi}}$$

$$\Rightarrow \quad \boxed{I_x = \frac{\pi R^4}{8}\left[1 - \frac{64}{9\pi^2}\right]}$$

Kural:



$$F = F_1 + F_2$$

$$I_x = I_x^{(1)} + I_x^{(2)}$$

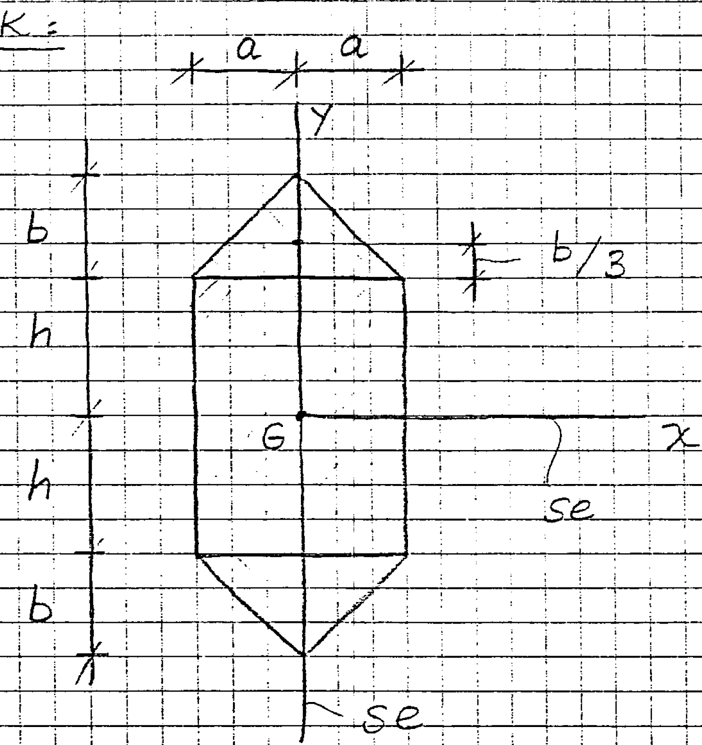
$$I_y = I_y^{(1)} + I_y^{(2)}$$

$$I_{xy} = I_{xy}^{(1)} + I_{xy}^{(2)}$$

ÖRNEK :

$$\left.\begin{matrix} I_x \\ I_y \\ I_{xy} \end{matrix}\right\} ?$$

$I_{xy} = 0$

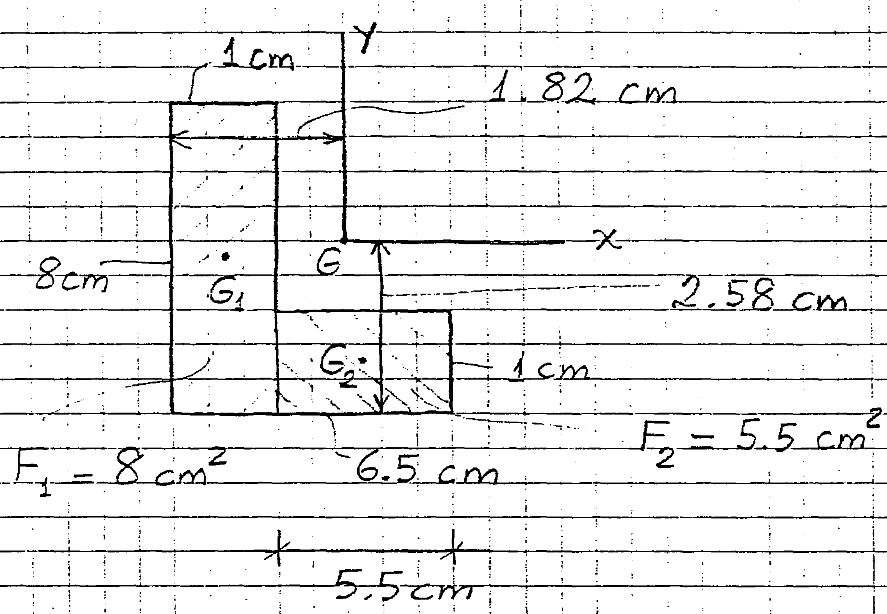


$G \rightarrow$ tüm kesit için ağırlık merkezi

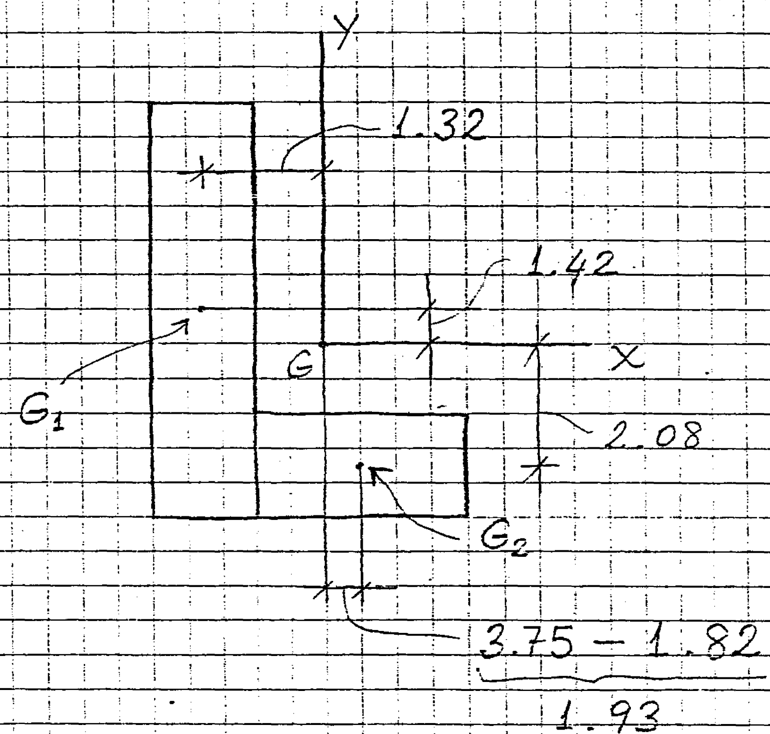
$$I_x = \frac{(2a)(2h)^3}{12} + 2 * \left[\frac{2ab^3}{36} + \left(h + \frac{b}{3}\right)^2 * \frac{2ab}{2} \right]$$

$$I_y = \frac{(2h)(2a)^3}{12} + 4 * \frac{ba^3}{12}$$

ÖRNEK:



$$\left.\begin{array}{l} I_x \\ I_y \\ I_{xy} \end{array}\right\} ?$$



$$I_x = \frac{1*8^3}{12} + (+1.42)^2*8 + \frac{5.5*1^3}{12} + (-2.08)^2*5.5$$

$$\Rightarrow \boxed{I_x \cong 83.1 \text{ cm}^4}$$

$$I_y = \frac{8*1^3}{12} + (-1.32)^2*8 + \frac{1*5.5^3}{12} + (+1.93)^2*5.5$$

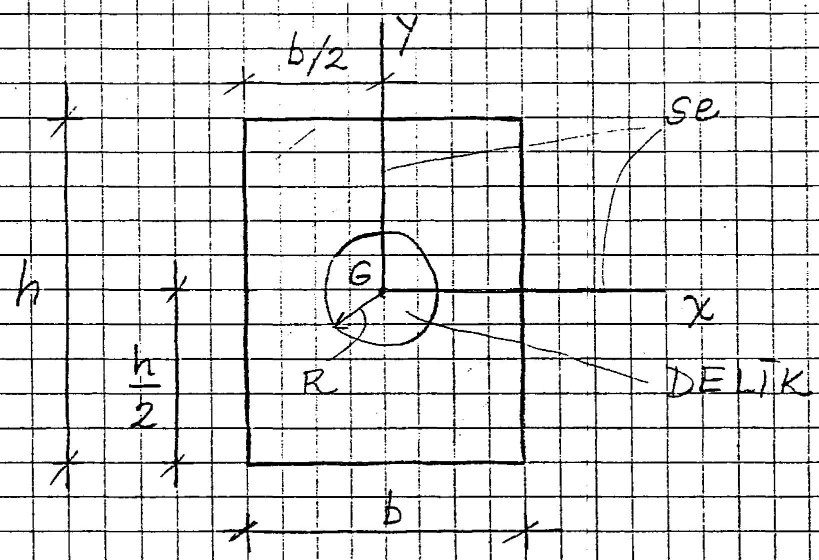
$$\Rightarrow \boxed{I_y \cong 49 \text{ cm}^4}$$

$$I_{xy} = 0 + (-1.32)(+1.42)*8 + 0 + (+1.93)(-2.08)*5.5$$

$$\Rightarrow \boxed{I_{xy} \cong -37.1 \text{ cm}^4}$$

Kural:

Kesitte delik var ise dolu kesitin atalet momentinden deliğin atalet momenti çıkarılır.



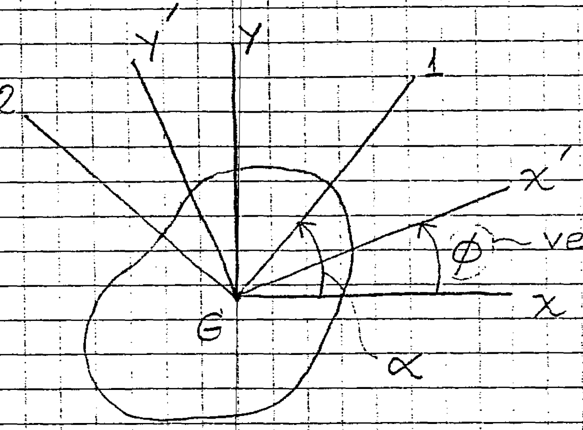
$$I_x = \frac{bh^3}{12} - \frac{\pi R^4}{4}$$

$$I_y = \frac{hb^3}{12} - \frac{\pi R^4}{4}$$

$$I_{xy} = 0$$

EKSENLERİN DÖNDÜRÜLMESİ

xy-nin orijini "G" de



$$\left.\begin{array}{l} I_x \\ I_y \\ I_{xy} \end{array}\right\}$$ hesap edilmiş ⇒ biliniyor

$$\left.\begin{array}{l} I_{x'} \\ I_{y'} \\ I_{x'y'} \end{array}\right\}$$? Bunun için dönüşüm formüllerini kullan:

$$I_{x'} = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\phi - I_{xy} \sin 2\phi$$

$$I_{y'} = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cos 2\phi + I_{xy} \sin 2\phi$$

$$I_{x'y'} = + \frac{I_x - I_y}{2} \sin 2\phi + I_{xy} \cos 2\phi$$

Çarpım atalet momentinin sıfır olduğu eksen takımına kesitin asal eksen takımı denir.

12 eksen takımı → asal eksen takımı

1,2 eksenleri → asal eksenler

$\Rightarrow I_{12} = 0$

I_1, I_2 ASAL ATALET MOMENTLERİ

>0

$$Tan\,2\alpha = \frac{-2I_{xy}}{I_x - I_y} \longrightarrow 2\alpha \; +\pi \;?$$

$$\Rightarrow \alpha$$

$$Sin\,2\alpha = \frac{-I_{xy}}{r}$$

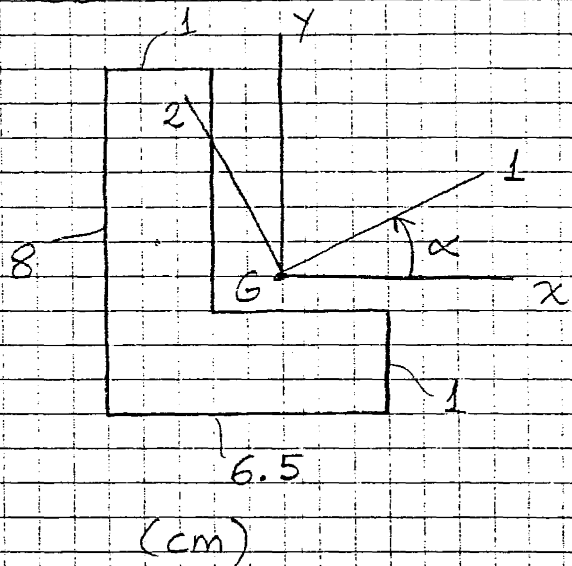
$$r = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_1 = \frac{I_x + I_y}{2} + r$$

$$I_2 = \frac{I_x + I_y}{2} - r$$

ÖRNEK:

önceki örnek:



$$\Rightarrow \begin{cases} I_x \cong 83.1 \text{ cm}^4 \\ I_y \cong 49 \text{ cm}^4 \\ I_{xy} \cong -37.1 \text{ cm}^4 \end{cases}$$

Asal Eksenler ?

Asal Atalet Momentleri ?

$$r = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} = 40.8 \text{ cm}^4$$

$$\tan 2\alpha = \frac{-2I_{xy}}{I_x - I_y} \cong 2.18$$

$$\Rightarrow 2\alpha \cong 65.4^\circ + \pi \;?$$

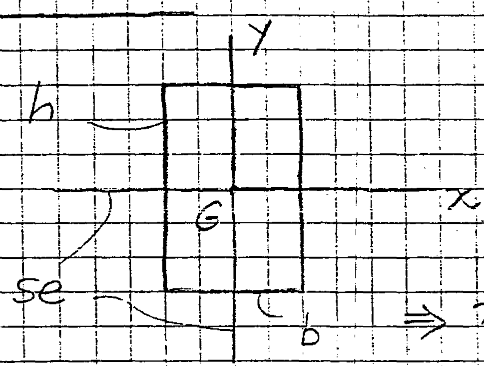
$$\sin 2\alpha = \frac{-I_{xy}}{r} > 0 \Rightarrow +\pi$$

$$\Rightarrow \boxed{\alpha = 32.7^\circ}$$

$$I_{12} = 0$$

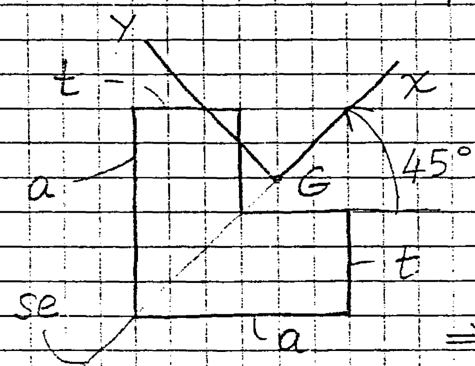
$$I_1 = \frac{I_x + I_y}{2} + r = 106.9 \text{ cm}^4$$

$$I_2 = \frac{I_x + I_y}{2} - r = 25.3 \text{ cm}^4$$



$I_{xy} = 0$

$\Rightarrow$ xy eksenleri asal eksenler



$I_{xy} = 0$

$\Rightarrow$ xy eksenleri asal eksenler