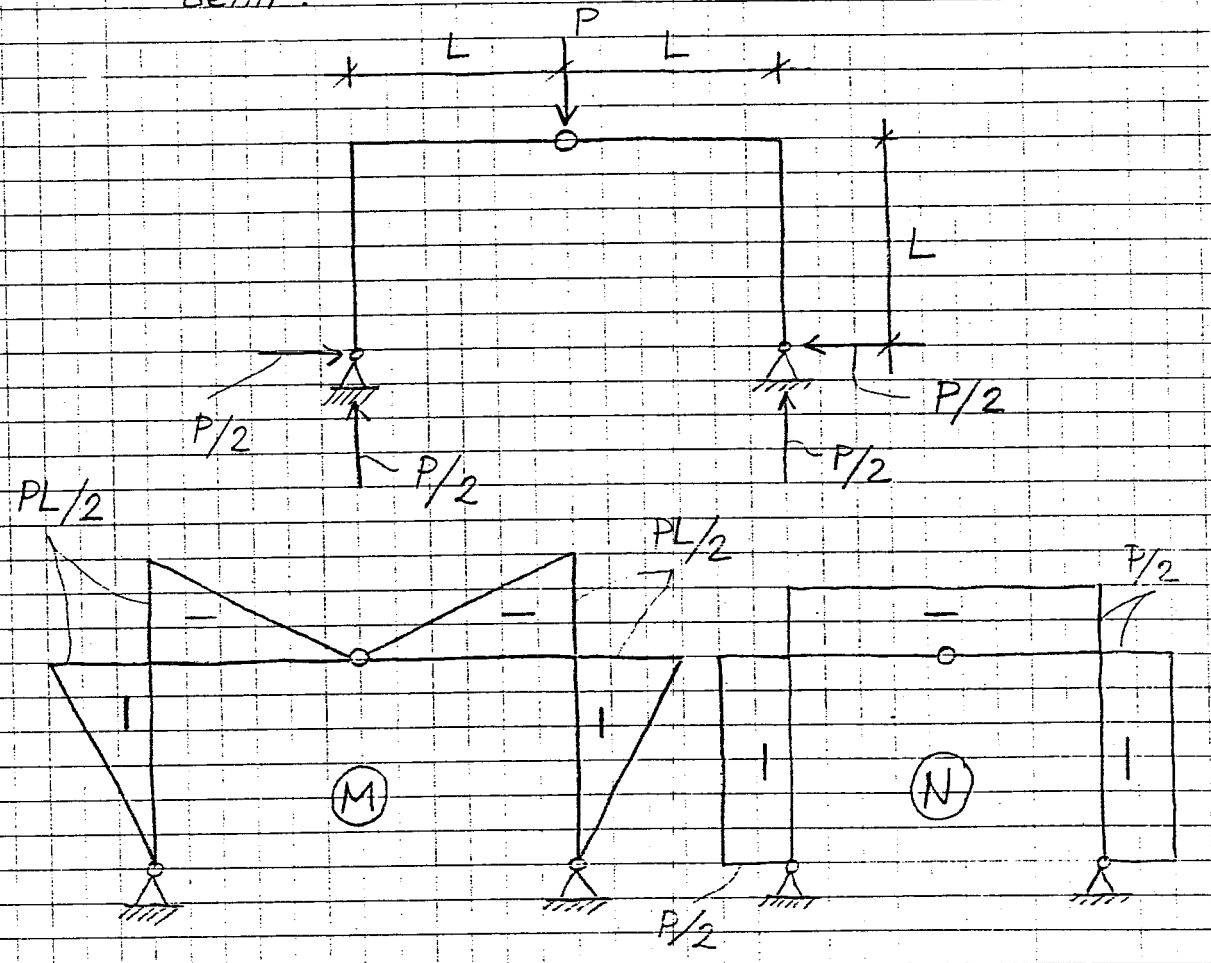
BÖLÜM 4

NORMAL KUVVETLİ EĞİLME

Hem normal kuvvet hem eğilme momenti etkisi altında meydana gelen eğilmeye normal kuvvetli eğilme denir.

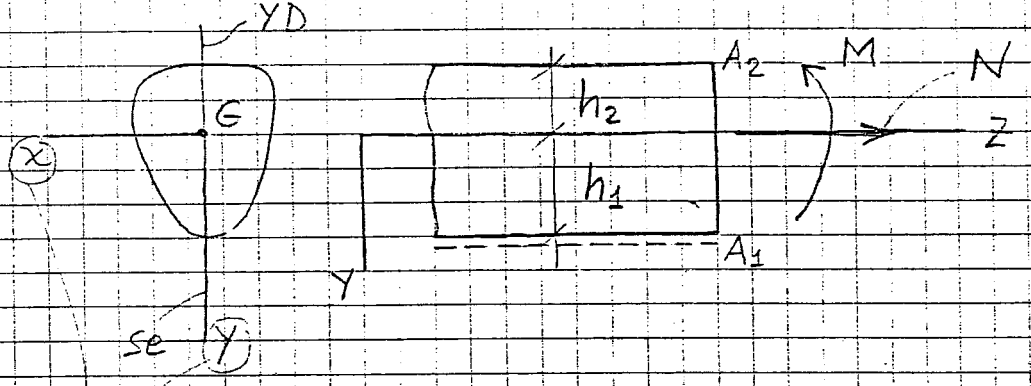


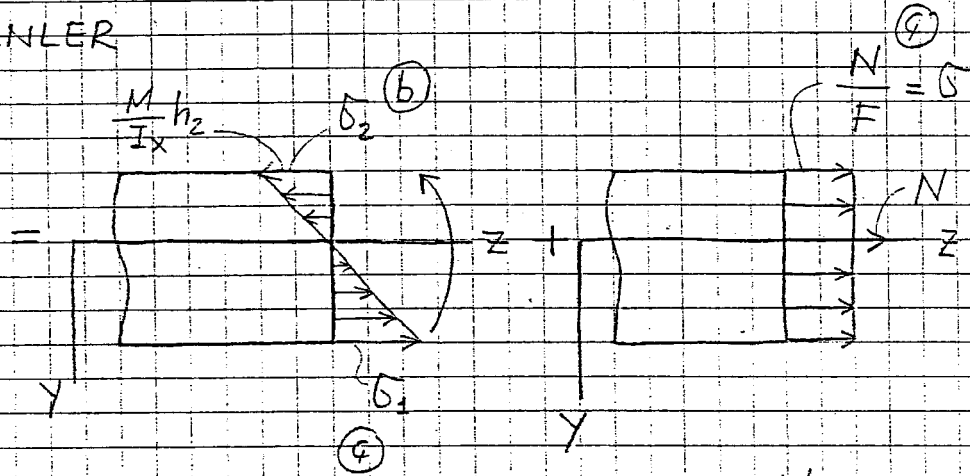
Bütün elemanlar normal kuvvetli eğilmeye maruz

Gerilme Analizi:

Kabuller:

- YD → düşey
- Kesit düşey eksen etrafında simetrik





$$\sigma = \frac{M}{I_x} y \qquad \sigma = \frac{N}{F}$$

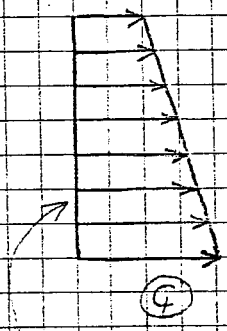
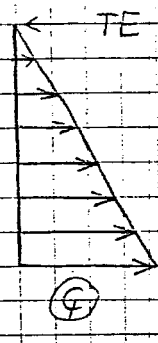
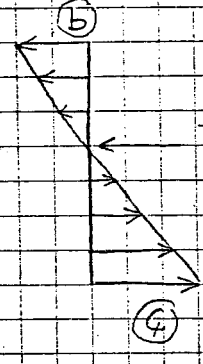
$$\sigma_2 = \sigma \Big|_{A_2} = \sigma \Big|_{y=-h_2} = -\frac{M}{I_x} h_2$$

$$\sigma = \frac{M}{I_x} y + \frac{N}{F} \quad (*)$$

σ: Toplam normal gerilme

$\frac{N}{F}$: DOĞRUSAL

Toplam Gerilme Şekli:

$\frac{N}{F} > \frac{M}{I_x} h_2$	$\frac{N}{F} = \frac{M}{I_x} h_2$	$\frac{N}{F} < \frac{M}{I_x} h_2$
		
TE kesit dışında		

TE'de $\sigma = 0$

TE'nin "y"si:

$\sigma = 0$

$$(*) \Rightarrow \frac{N}{F} + \frac{M}{I_x} y = 0$$

$$\Rightarrow \quad y = -\frac{N}{F}\,\frac{I_x}{M}$$

$$= -\frac{I_x/F}{M/N} \qquad I_x/F \rightarrow i_x^2$$

$M/N \rightarrow e$ eksantrisite

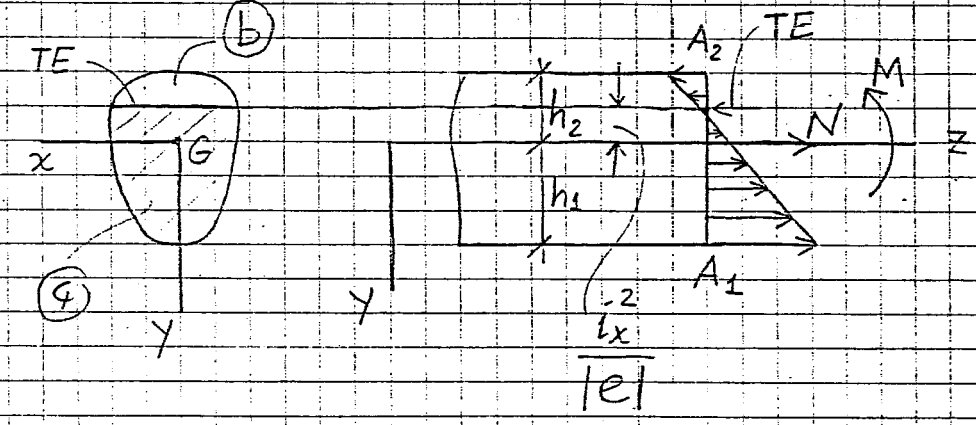
$$i_x = \sqrt{\frac{I_x}{F}}$$

$$\Rightarrow \quad \boxed{y = -\frac{i_x^2}{e}}$$

y: TE nin "y" si

$\sigma = 0$

$$i_x = \sqrt{I_x/F} > 0 \quad , \quad e = \frac{M}{N} \gtrless 0$$



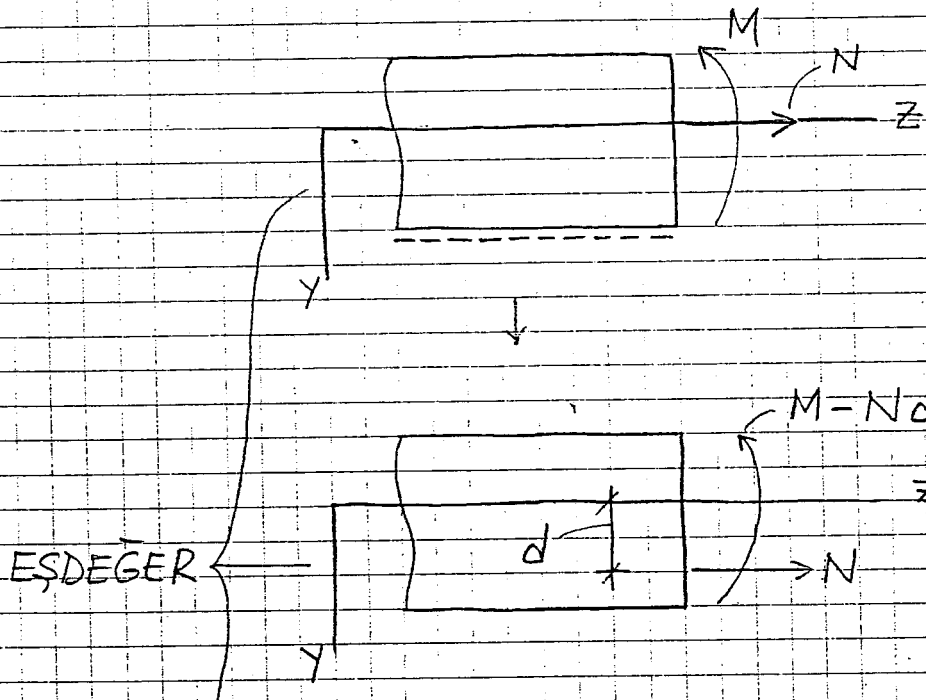
$$\sigma = \frac{M}{I_x}y + \frac{N}{F}$$

y ⊕ → aşağı doğru

y ⊖ → yukarı doğru

$e = \frac{M}{N}$ için bir açıklama:

e → eksantrisite

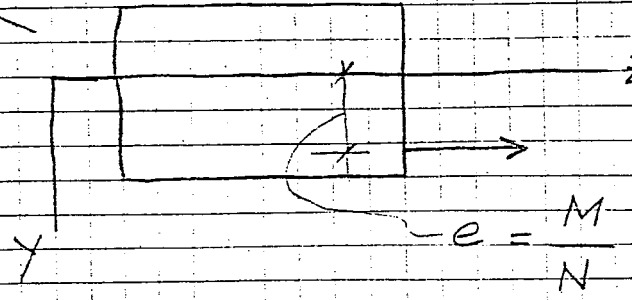


Momenti sıfır yapan "d" mesafesi:

$M - Nd = 0$

$\Rightarrow d = \boxed{\frac{M}{N} = e}$

eksantrisite



$$[e] = \frac{[M]}{[N]} = \frac{FL}{F}$$

$$\Rightarrow [e] = L$$

$$[i_x] = L$$

$$y = -\frac{i_x^2}{e} \Rightarrow [y] = \frac{L^2}{L} = L$$

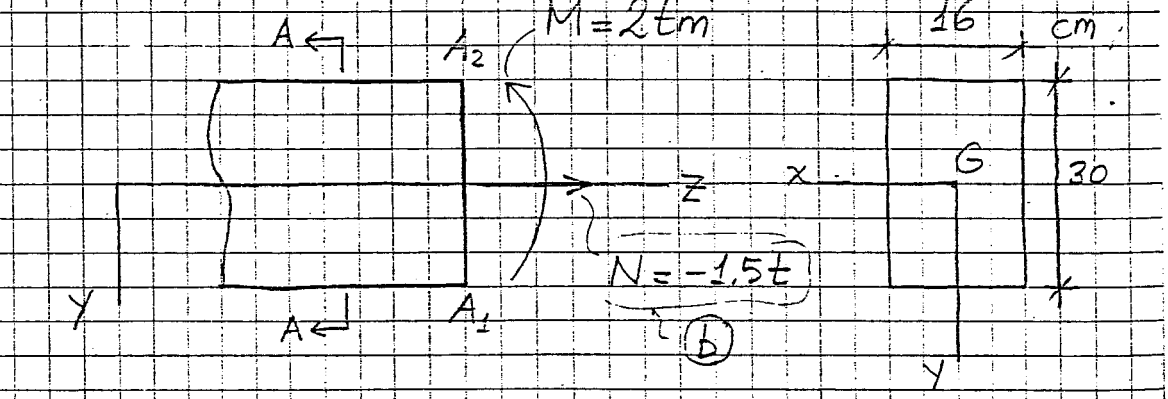
$e \gtrless 0$

$\frac{M}{N}$

$e \rightarrow \oplus \Rightarrow$ aşağı doğru

$e \rightarrow \ominus \Rightarrow$ yukarı doğru

ÖRNEK:



Gerilme dağılımı ?

$$\sigma = \frac{N}{F} + \frac{M}{I_x} y$$

$N = -1500$ kgf, $M = 2\times10^5$ kgf-cm

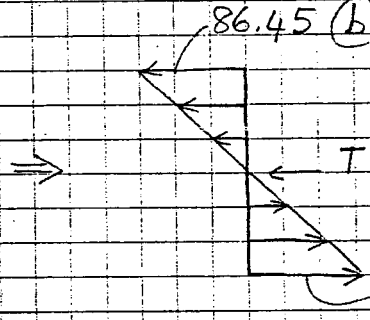
$F = 480\,cm^2$, $I_x = \frac{16\times30^3}{12} = 36000\,cm^4$

$$\sigma_1 = \sigma\Big|_{y=+15cm} = -\frac{1500}{480} + \frac{2\times10^5}{36000}(+15)$$

$$\simeq 80.27\ kgf/cm^2 \quad (c)$$

$$\sigma_2 = \sigma\Big|_{y=-15} = -\frac{1500}{480} + \frac{2\times10^5}{36000}(-15)$$

$$\simeq -86.45\ kgf/cm^2 \quad (b)$$

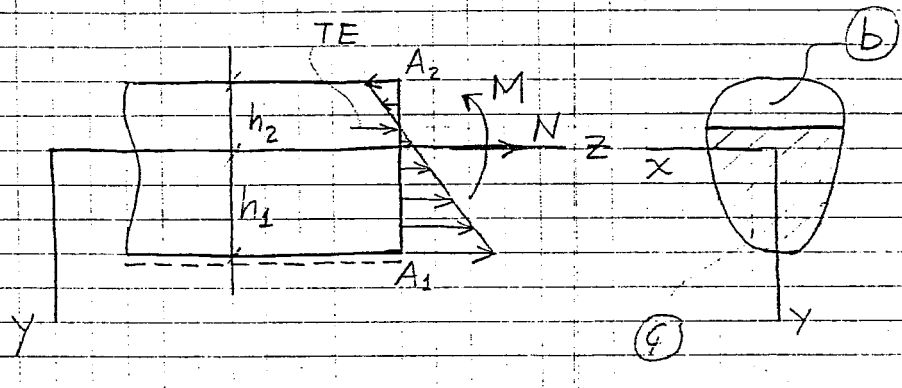


$$y = -\frac{i_x^2}{e}, \quad \frac{I_x}{F} = \frac{36000}{480}$$

$$e = \frac{M}{N} = \frac{2\times10^5}{-1500} = -133.33\ cm$$

$$\Rightarrow y = +0.56\ cm$$

BOYUTLANDIRMA



$$\sigma = \frac{N}{F} + \frac{M}{I_x} y$$

$\sigma_{çmax}$ → mutlak (ç) gerilmesi

σ_{bmax} → mutlak (b) gerilmesi

x-ekseninden en uzak noktalarda oluşur.

a) $\sigma_{em} \neq \sigma'_{em}$ ise:

$$\Rightarrow \boxed{\begin{aligned} \sigma_{çmax} &\leq \sigma_{em} \\ \sigma_{bmax} &\leq \sigma'_{em} \end{aligned}}$$

b) $\sigma_{em} = \sigma'_{em}$ ise:

$$\Rightarrow \sigma_{max} = \max(\sigma_{bmax}, \sigma_{cmax})$$

$$\Rightarrow \boxed{\sigma_{max} \leq \sigma_{em}}$$

- ÖZEL HAL:

$$\begin{cases} \sigma_{em} = \sigma'_{em} \\ h_1 = h_2 = h' \end{cases}$$

$$\Rightarrow W_x = \frac{I_x}{h'}$$

$$\sigma_1 = \sigma\Big|_{y=h_1} = \frac{N}{F} + \frac{M}{I_x} h_1$$

$$\sigma_2 = \sigma\Big|_{y=-h_2} = \frac{N}{F} + \frac{M}{I_x}(-h_2)$$

$h_1 = h_2 = h'$ ise:

$$\begin{cases} \sigma_1 = \dfrac{N}{F} + \dfrac{M}{I_x} h' \\ \sigma_2 = \dfrac{N}{F} - \dfrac{M}{I_x} h' \end{cases}$$

$$\Rightarrow \sigma_1 = \frac{N}{F} + \frac{M}{I_x/h'}$$

W_x

$$\sigma_2 = \frac{N}{F} - \frac{M}{I_x/h'}$$

$$\Rightarrow \sigma_1 = \frac{N}{F} + \frac{M}{W_x}$$

$$\sigma_2 = \frac{N}{F} - \frac{M}{W_x}$$

$$\Rightarrow \sigma_{max} = \max(|\sigma_1|, |\sigma_2|)$$

$$\Rightarrow \boxed{\sigma_{max} = \max\left|\frac{N}{F} \mp \frac{M}{W_x}\right|}$$

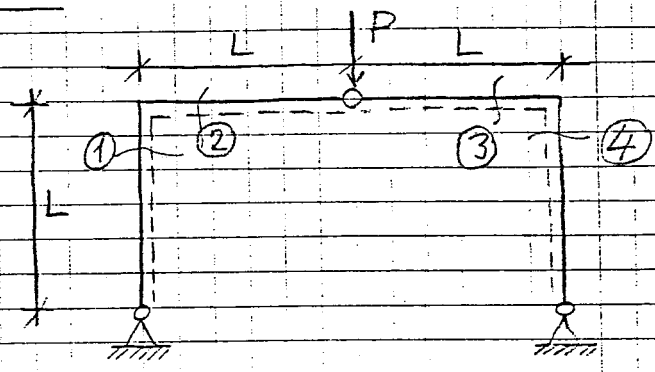
$\max|\sigma|$ — hangi işaret mutlak max değeri verirse o işaret alınacak

$$\Rightarrow \sigma_{max} = \boxed{\max\left|\frac{N}{F} \mp \frac{M}{W_x}\right| \leq \sigma_{em}}$$

$\sigma_{em} = \sigma'_{em}$ } ise geçerli

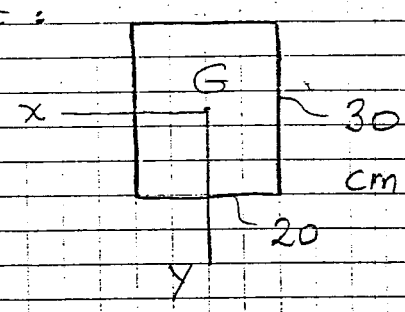
$h_1 = h_2$ }

ÖRNEK:



$L = 2\,m$

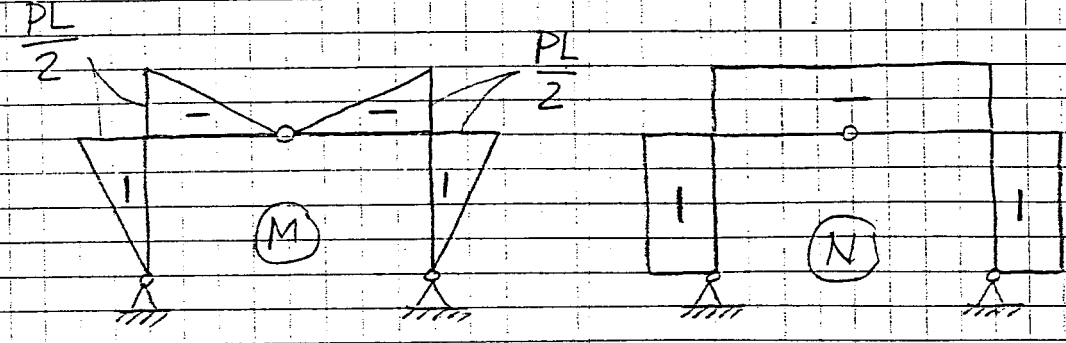
kesit:



$\sigma_{em} = \sigma'_{em} = 200\ kgf/cm^2$

$P_{max} = ?$

└ Çerçevenin emniyetle taşıyabileceği max P yükü

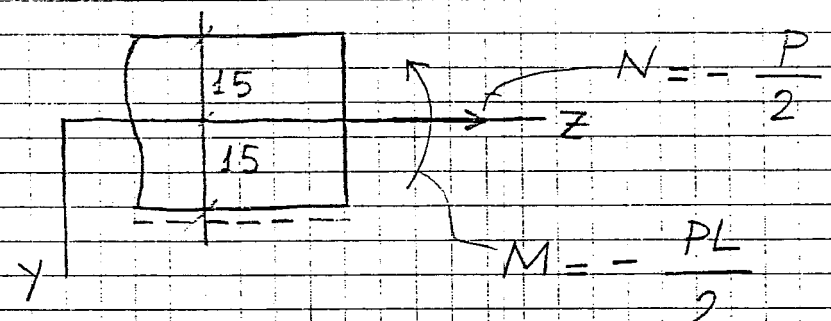


max M ve max N'nin etki ettiği kesitler: ①, ②, ③, ④

② kesitini al:

L → cm.

F → kgf (kuvvet)



$h_1 = h_2 = 15$ cm

$\sigma_{em} = \sigma'_{em} = 200$ kgf/cm²

$N = -\frac{P}{2}$ kgf

$M = -\frac{PL}{2} = -100P$ kgf-cm (L = 200 cm)

$F = 600$ cm²

$W_x = \frac{bh^2}{6} = \frac{(20)(30)^2}{6} = 3000$ cm³

$$\max \left| \frac{N}{F} + \frac{M}{W_x} \right|$$

$N = -0.5P$, $M = -100P$, $F = 600$, $W_x = 3000$

⊕ alınacak

$$\Rightarrow 0.0341P \leq 200$$

$$\Rightarrow P \leq 5865 \text{ kgf}$$

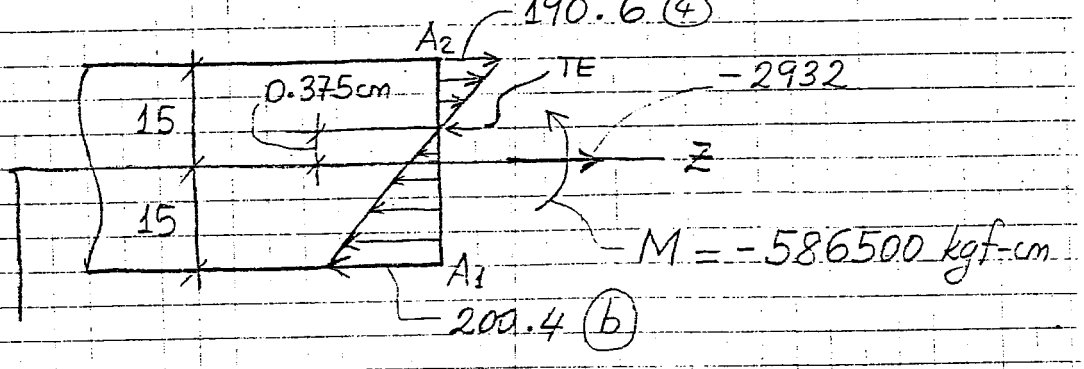
$$\Rightarrow \boxed{P_{max} = 5865 \text{ kgf}}$$

P_{max} için ② kesitindeki gerilme dağılışı:

$$P = P_{max} = 5865 \text{ kgf}$$

$$\Rightarrow N = -\frac{P}{2} = -2932 \text{ kgf}$$

$$M = -100P = -586500 \text{ kgf-cm}$$



$$\sigma = \frac{N}{F} + \frac{M}{I_x} y$$

$$\sigma_1 = \sigma\Big|_{y=+15} = \frac{-2932}{600} + \frac{-586500}{\frac{20*30^3}{12}} (+15)$$

$$\Rightarrow \boxed{\sigma_1 = -200.4 \ kgf/cm^2} \ (b)$$

$$\sigma_2 = \sigma\Big|_{y=-15} = \frac{-2932}{600} + \frac{-586500}{\frac{20*30^3}{12}} (-15)$$

$$\Rightarrow \boxed{\sigma_2 = +190.6 \ kgf/cm^2} \ (c)$$

TE'nin yeri:

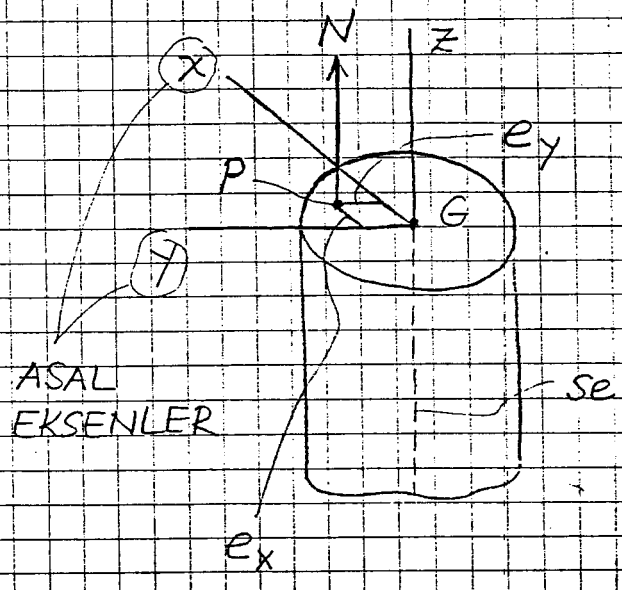
$$y = -\frac{i_x^2}{e}$$

$i_x^2 = \frac{I_x}{F} = 75 cm^2$

$e = \frac{M}{N} = \frac{-586500}{-2932}$

$$\Rightarrow \boxed{y = -0.375 \ cm}$$

EKSANTRİK KUVVET HALİ



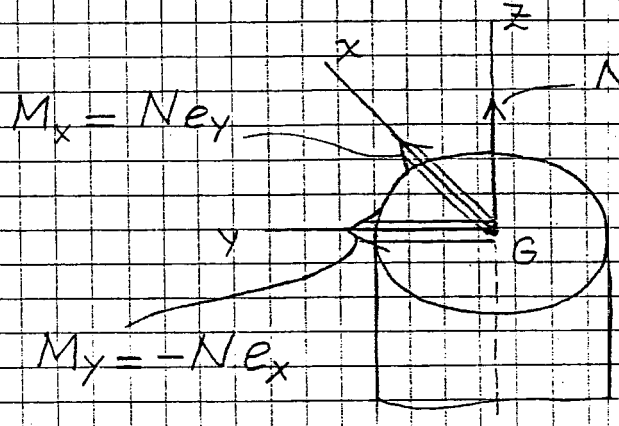
$P \rightarrow (e_x, e_y)$

P'nin koordinatları

e_x, e_y → eksantrisite

x-yönünde y-yönünde

N'yi "G"ye kaydır:



Hatırla:

Eğik Eğilme:

$$\Rightarrow \sigma = \frac{M_x}{I_x} y - \frac{M_y}{I_y} x$$

$$\Rightarrow \sigma = \frac{N}{F} + \frac{M_x}{I_x} y - \frac{M_y}{I_y} x$$

$M_x = N e_y$, $\quad -M_y = -N e_x$

$$= \frac{N}{F} + \frac{N e_y}{I_x} y + \frac{N e_x}{I_y} x$$

$$= \frac{N}{F} \left(1 + \frac{e_y}{I_x/F} y + \frac{e_x}{I_y/F} x \right)$$

$I_x/F = i_x^2$, $\quad I_y/F = i_y^2$

$$\Rightarrow \boxed{\sigma = \frac{N}{F} \left(1 + \frac{e_y}{i_x^2} y + \frac{e_x}{i_y^2} x \right)}$$