$\Rightarrow$ DE: $\sigma = \frac{N}{A} \leq \sigma_a$ ← DE

$$\Rightarrow \boxed{A \geq \frac{N}{\sigma_a}}$$

2) Checking the dimensions of CS.

knowns	question
N	Is CSA sufficient ?
A	
σ_a	

DE: $\sigma = \frac{N}{A} \overset{?}{\leq} \sigma_a$

3) Determination of load capacity of CS.

knowns	unknown
A	N
σ_a	

DE: $\sigma = \frac{N}{A} \leq \sigma_a$

$\Rightarrow \boxed{N \leq \sigma_a * A}$ N_{max} — the max. load which the CS can carry safely

Sometimes AS's might be different for tensile and compressional cases. When this is the case, we will have two different AS's, one is tensile AS, the other is compressional AS.

σ_a → AS for tension (T) — TAS

σ_a' → AS for compression (C) — CAS

> 0 — material properties

For example for concrete:

$$\sigma_a' >> \sigma_a$$

VERY LARGE

When the material has two different AS's for T and C cases, DE must be modified as follows:

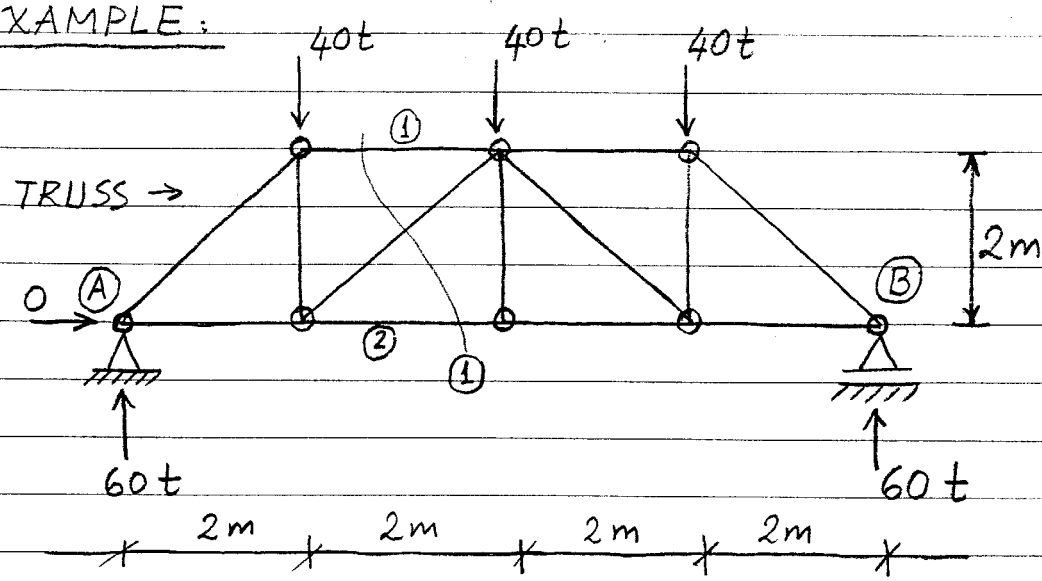
When $\overset{\sigma}{NS} > 0 \Rightarrow \sigma_{max} \leq \sigma_a$

σ_{max}: max. tensile stress

When $NS < 0 \Rightarrow \sigma_{max} \leq \sigma_a'$

σ_{max}: absolutely max. compressional stress) $\max|\sigma|$

EXAMPLE:



F → kgf

L → cm

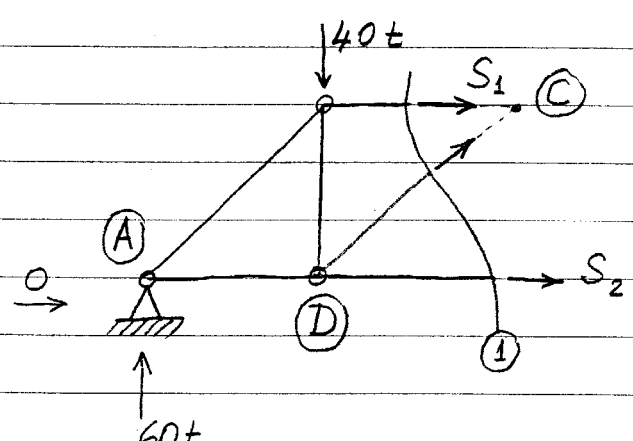
Given: $\sigma_a' = 1000\ kgf/cm^2$

$\sigma_a = 2000\ kgf/cm^2$

we wish to find CSA's of the bars ① & ②

Axial forces in ① & ②:

Section method:



$+\circlearrowleft \sum M_C = 0$

$\Rightarrow S_2 * 2 + 40 * 2 - 60 * 4 = 0$

$$\Rightarrow \boxed{S_2 = 80\ t\ (T)}$$

$$\overset{+}{\circlearrowleft} \sum M_{(D)} = 0 \Rightarrow 60 * 2 + S_1 * 2 = 0$$

$$\Rightarrow \boxed{S_1 = -60\ t\ (C)}$$

Bar (1):

$$S_1 = -60\ t\ (C)$$

DE:

$$|\sigma| = \frac{|S_1|}{A} = \frac{60000\ kgf}{A} \leq \sigma_a' \quad (1000)$$

$$\Rightarrow A \geq 60\ cm^2$$

$$\Rightarrow \text{CHOOSE}: A = 80\ cm^2$$

Bar (2):

$$S_2 = 80\ t\ (T)$$

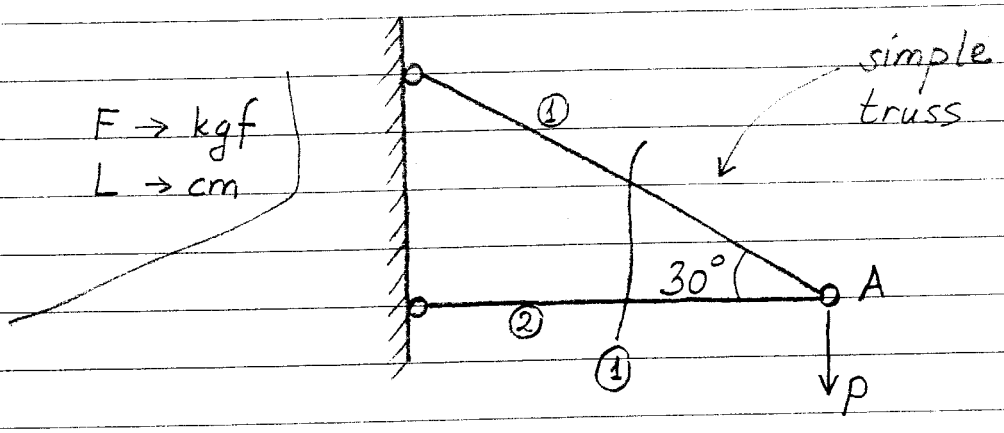
DE:

$$\sigma = \frac{S_2}{A} = \frac{80000\ kgf}{A} \leq \sigma_a \quad (2000)$$

$$\Rightarrow A \geq 40\ cm^2$$

$$\Rightarrow \text{CHOOSE}: A = 60\ cm^2$$

EXAMPLE:



Given:

$\sigma_a' = 500\ kgf/cm^2$

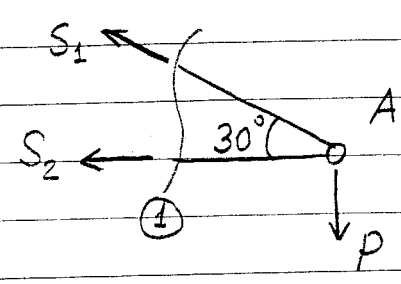
$\sigma_a = 1000\ kgf/cm^2$

$A_1 = 50\ cm^2,\ A_2 = 30\ cm^2$

We wish to find the max. value of P which the system can carry SAFELY.

Find the axial f. in ① & ② in terms of P:

Joint method:



$$+\uparrow \sum F_y = 0 \Rightarrow S_1 * \underbrace{Sin\,30^\circ}_{0.5} - P = 0$$

$$\Rightarrow \boxed{S_1 = 2P \quad (T)}$$

$$+\leftarrow \sum F_x = 0 \Rightarrow S_2 + S_1 \cos 30^\circ = 0$$

$S_1 = 2P$, $\cos 30^\circ = \sqrt{3}/2$

$$\Rightarrow \boxed{S_2 = -\sqrt{3}\,P \quad (C)}$$

Bar (1): $\Rightarrow$ (T)

DE: $\sigma = \dfrac{S_1}{A} \leq \sigma_a$

$S_1 = 2P$, $A = 50$, $\sigma_a = 1000$

$$\Rightarrow \boxed{P \leq 25000 \text{ kgf} \quad (*)}$$

$25000 \text{ kgf} = 25 \text{ t}$

Bar (2): $\Rightarrow$ (C)

DE: $|\sigma| = \dfrac{|S_2|}{A} \leq \sigma_a'$

$|S_2| = \sqrt{3}P$, $A = 30$, $\sigma_a' = 500$

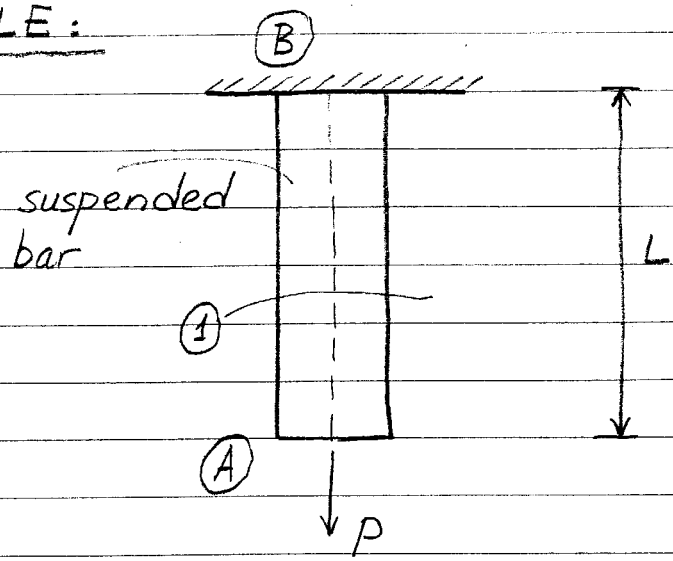
$$\Rightarrow \boxed{P \leq 8660 \text{ kgf} \quad (**)}$$

(*) & (**) must be satisfied.

$$\Rightarrow \boxed{P_{max} = 8660 \text{ kgf}}$$

max. value of P which the sys. can cary safely

EXAMPLE:



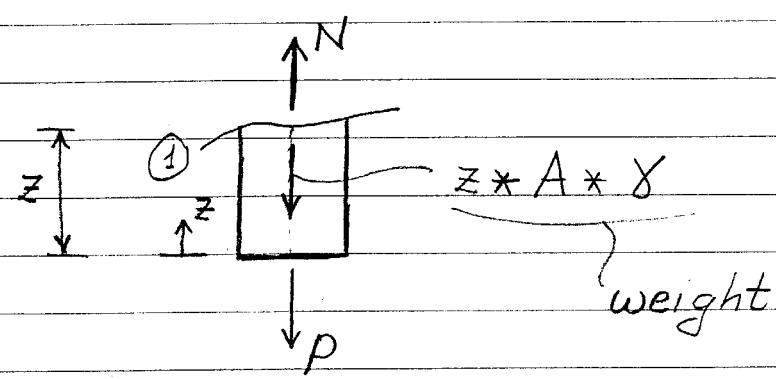
Given:

$$L, P, \sigma_a, \gamma$$

γ: specific weight — weight per unit volume

we wish to find CSA.

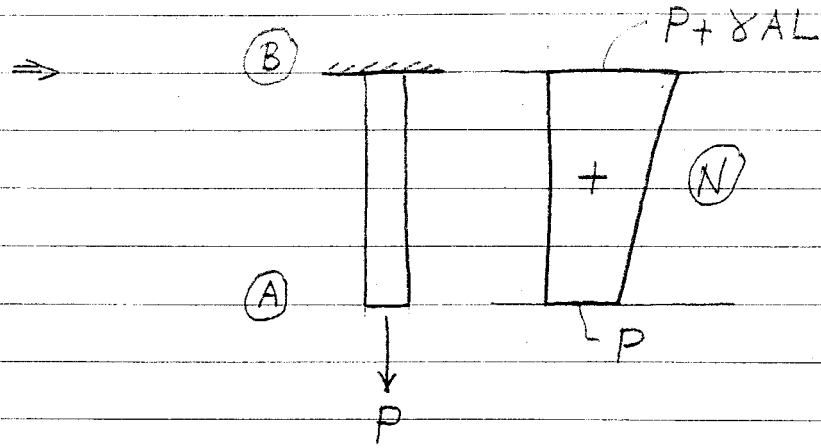
Axial load in the bar:



$$+\uparrow \sum F_z = 0 \Rightarrow N - P - \gamma A z = 0$$

$$\Rightarrow N = P + \gamma A z \rightarrow \text{LINEAR}$$

(A) $\Rightarrow z = 0 \Rightarrow N = P$ (B) $\Rightarrow z = L \Rightarrow N = P + \gamma AL$



$$\Rightarrow N_{max} = P + \gamma AL \quad (T)$$

DE:

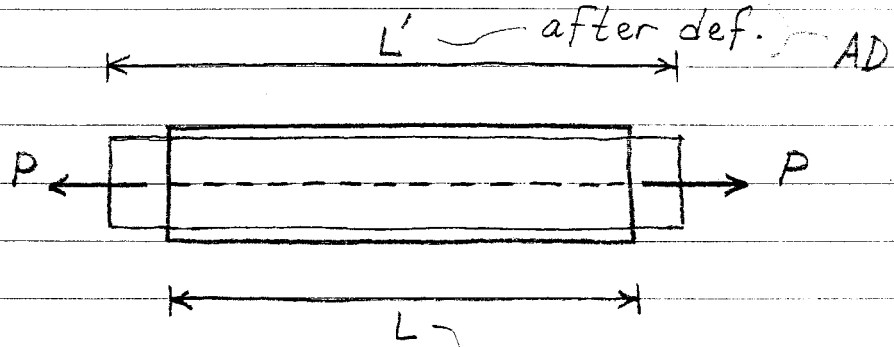
$$\sigma_{max} = \frac{N_{max}}{A} \leq \sigma_a$$

$$\Rightarrow \frac{P + \gamma AL}{A} \leq \sigma_a$$

$$\Rightarrow \frac{P}{A} + \gamma L \leq \sigma_a$$

$$\Rightarrow \boxed{A \geq \frac{P}{\sigma_a - \gamma L}}$$

DEFINITION OF STRAIN FOR A BAR SUBJECTED TO AN AXIAL LOAD



length change

$$\Delta L = L' - L$$

$$\varepsilon \overset{\Delta}{=} \frac{\Delta L}{L}$$

definition

← measure for the def. in axial dir.

ε: longitudinal strain

$\frac{\Delta L}{L}$: length change per unit length

LON. STN.

$$\Rightarrow \boxed{\varepsilon \overset{\Delta}{=} \frac{\Delta L}{L}}$$

If we know ε, we can find ΔL by:

$$\Delta L = \varepsilon * L$$

ΔL: length change; ε: lon. stn.; L: initial length

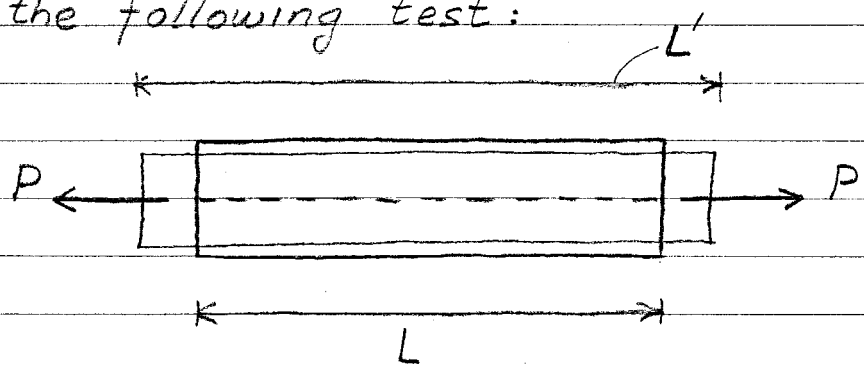
$$\varepsilon = \frac{\Delta L}{L} = \frac{L' - L}{L}$$

$\Rightarrow \varepsilon$ — $\oplus \Rightarrow \Delta L > 0 \Rightarrow$ length increase

$\ominus \Rightarrow \Delta L < 0 \Rightarrow$ length decrease

TENSION TEST

Suppose that in laboratory we do the following test:



Starting with $P=0$ we increase the load gradually, and we measure the length increase at each load level.

$$P=0 \rightarrow \underbrace{\Delta L = 0}_{\varepsilon = \frac{\Delta L}{L} = 0} \rightarrow \sigma = \frac{P}{A} = 0$$

(A: CSA)

$$P = P_1 \rightarrow \underbrace{\Delta L = \Delta L_1}_{\varepsilon_1 = \frac{\Delta L_1}{L}} \rightarrow \sigma = \frac{P_1}{A}$$

(ΔL_1: measured)

$$P = P_2 \rightarrow \Delta L = \Delta L_2 \rightarrow \varepsilon_2 = \frac{\Delta L_2}{L} \rightarrow \sigma = \frac{P_2}{A}$$

(ΔL_2: measured)

$\vdots$

CONTINUE