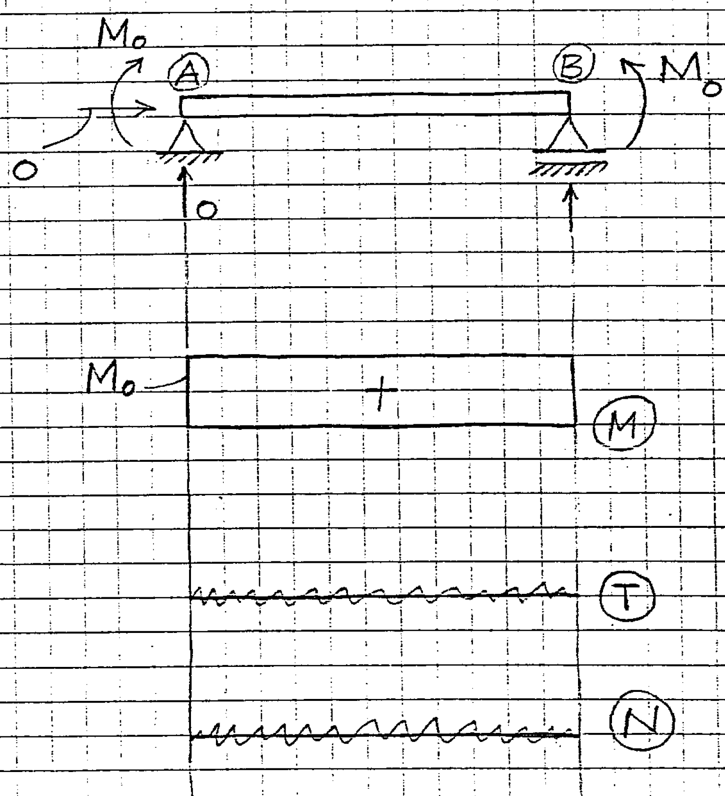
BÖLÜM 2

BASİT EĞİLME

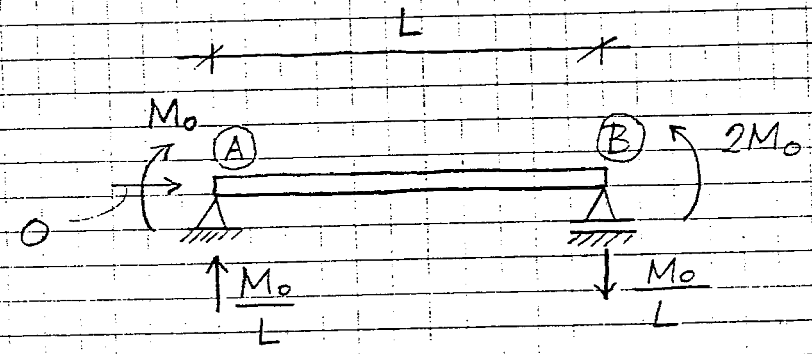
Sadece eğilme momentleri etkisi altındaki eğilmeye basit eğilme denir. Yani basit eğilmede normal kuvvet ve kesme kuvveti sıfırdır.

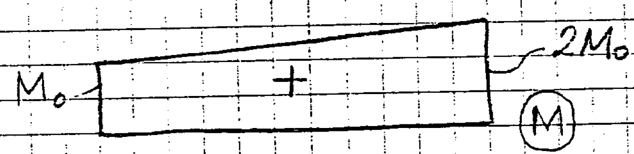


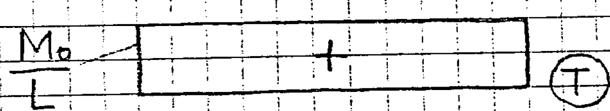
AB çubuğu için :

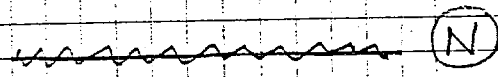
$$M = M_o \neq 0$$
$$N = T = 0$$

$\Rightarrow$ AB çubuğu basit eğilmeye maruz.

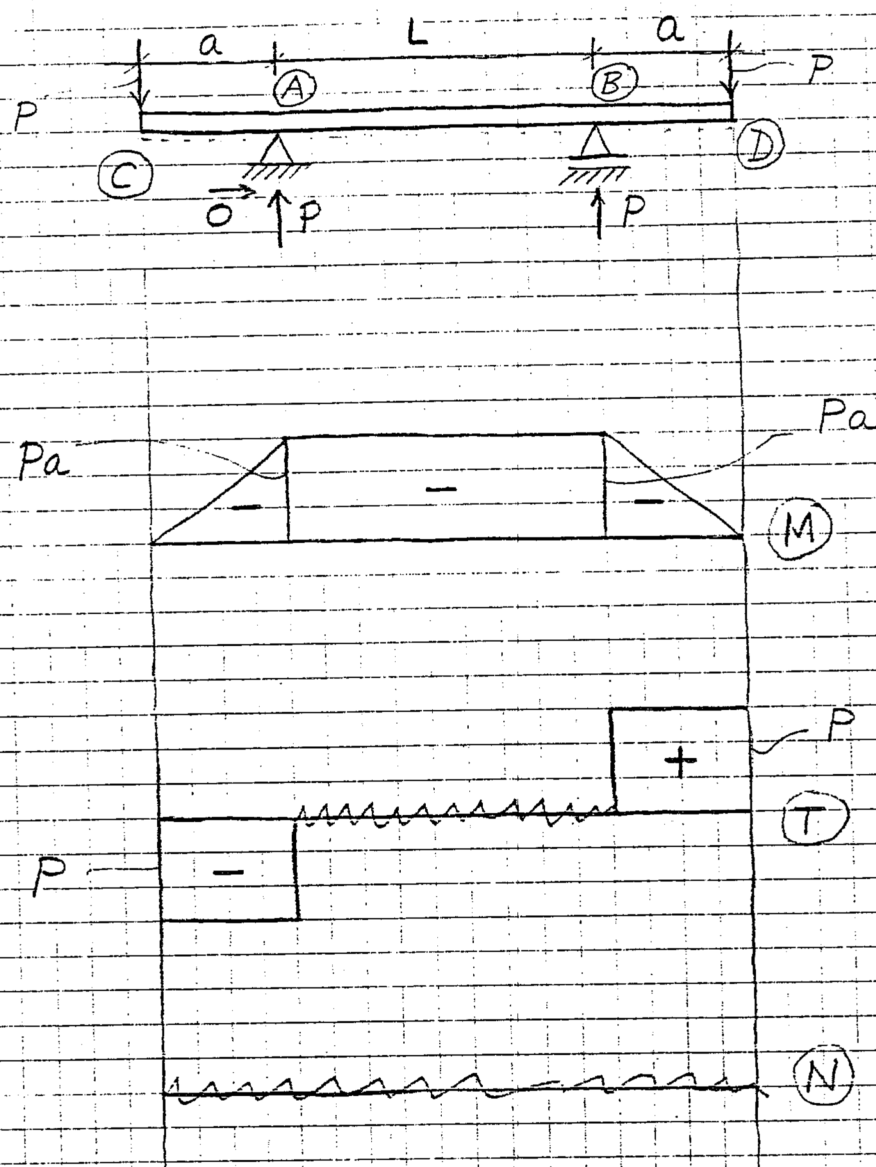








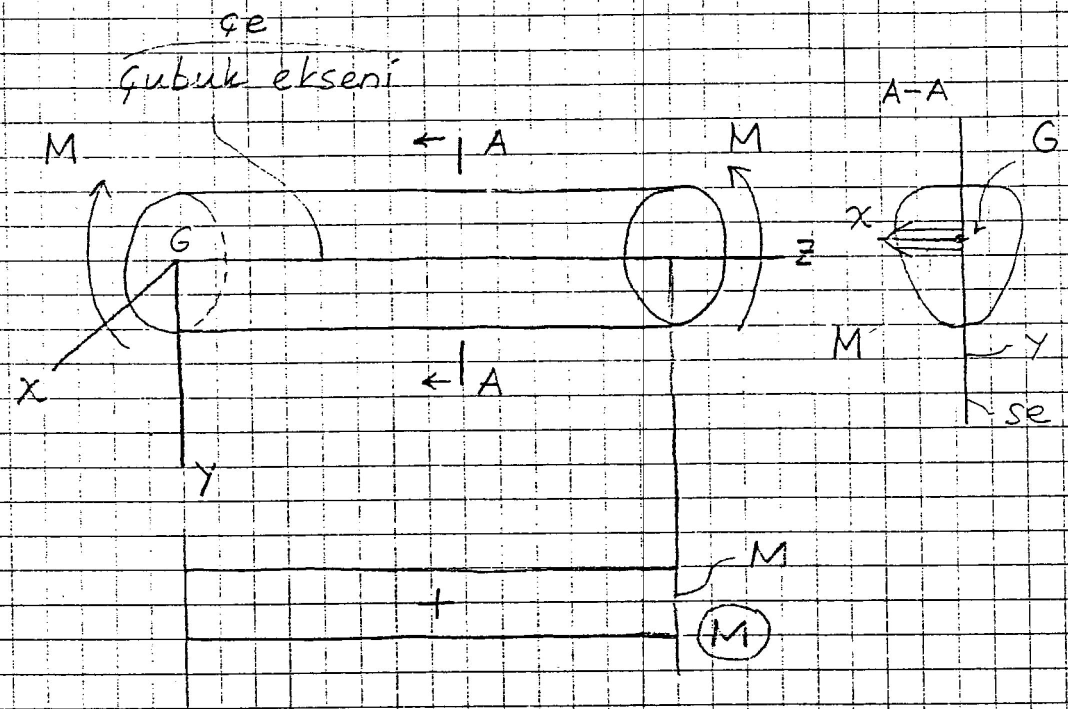
$$\left.\begin{array}{l} M \neq 0 \\ T \neq 0 \\ N = 0 \end{array}\right\} \Rightarrow \text{Eğilme Basit Değil}$$



AB → Basit Eğilmeye maruz

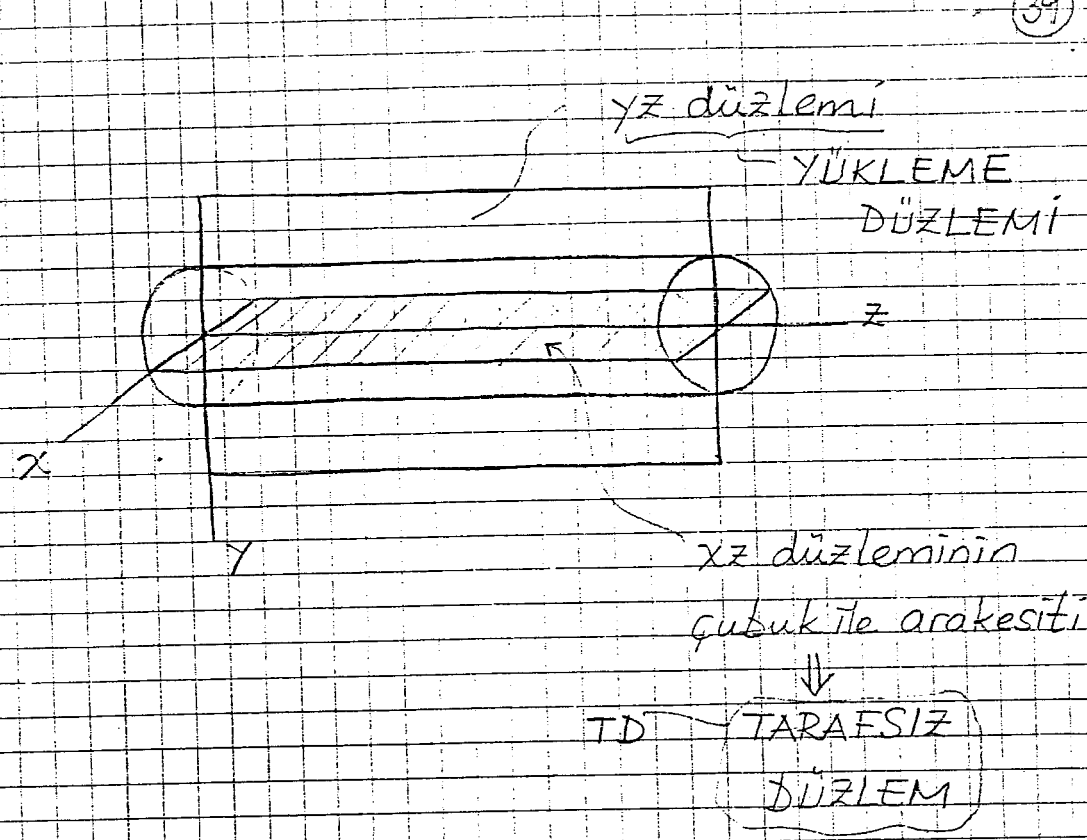
AC, BD → Kesmeli Eğilmeye maruz

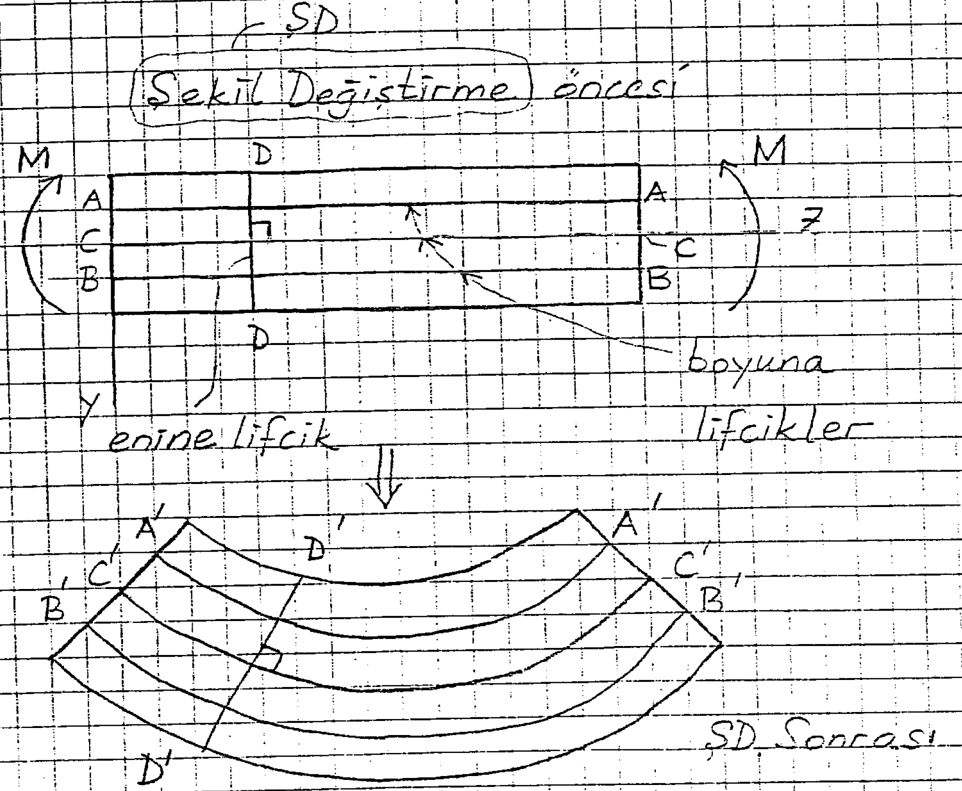
SİMETRİK KESİTLERİN BASİT EĞİLMESİ



Çubuk kesiti ağırlık merkezleri ile çakışan eksene çubuk ekseni denir.

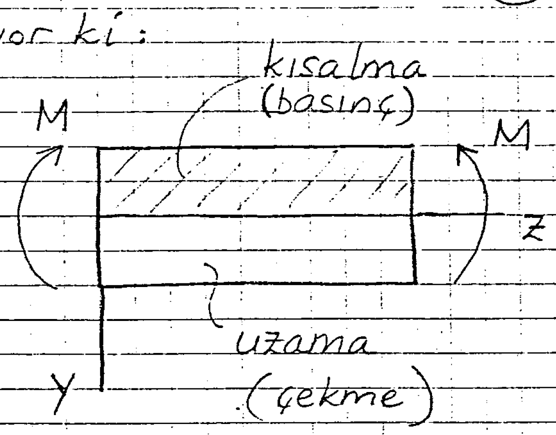
"M"nin döndürme etkisi yz düzleminde.


yz düzlemi
YÜKLEME DÜZLEMİ
z
x
y
xz düzleminin çubuk ile arakesiti
⇓
TD
TARAFSIZ DÜZLEM


ŞD
Şekil Değiştirme öncesi
M
D
A
C
B
M
A
C
B
z
D
y
boyuna lifcikler
enine lifcik
⇓
A'
D'
A'
C'
B'
C'
B'
D'
ŞD Sonrası

Deneyler gösteriyor ki:

$$\left.\begin{array}{l} \overline{C'C'} = \overline{CC} \\ \overline{B'B'} > \overline{BB} \\ \overline{A'A'} < \overline{AA} \end{array}\right\} \Rightarrow$$



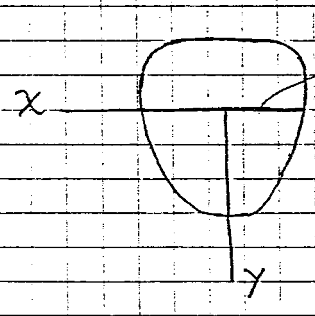
ŞD sonrası:

1) DD doğru kalıyor } Bernoulli
2) $D'D' \perp TD$ } Hipotezleri

Bernoulli Hipotezlerinin başka bir şekilde ifadesi:

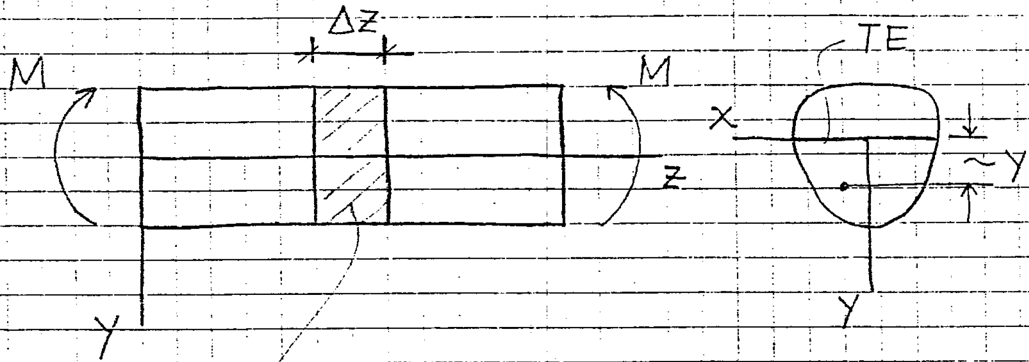
1) Dik kesitler eğilmeden sonra düzlem kalır

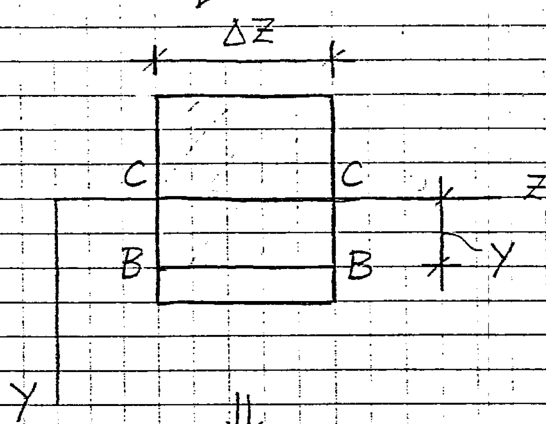
2) Dik kesitler eğilmeden sonra TD'ye dik kalır



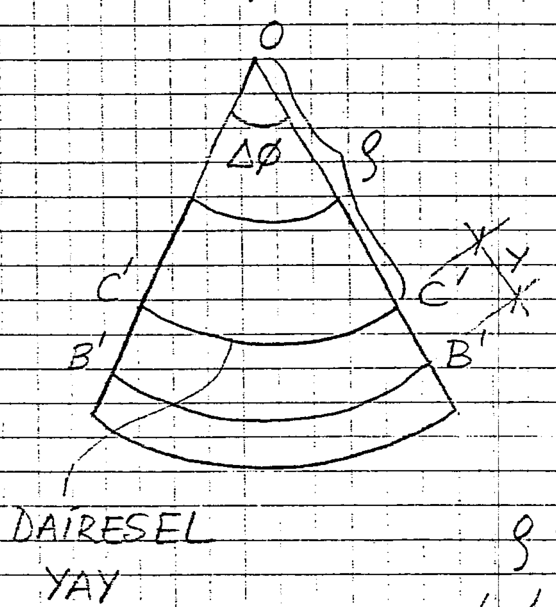
TD ile çubuğun arakesiti } TARAFSIZ EKSEN (TE)

BİRİM UZAMA İFADESİ





⇓



$\rho \rightarrow$ eğrilik yarıçapı

$\left(\dfrac{1}{\rho}\right) \rightarrow$ eğrilik

$$\underbrace{\overline{C'C'}}_{\rho\Delta\phi} = \underbrace{\overline{CC}}_{\Delta z}$$

$$\Rightarrow \boxed{\Delta z = \rho\Delta\phi}$$

BB lifciği için birim uzama:

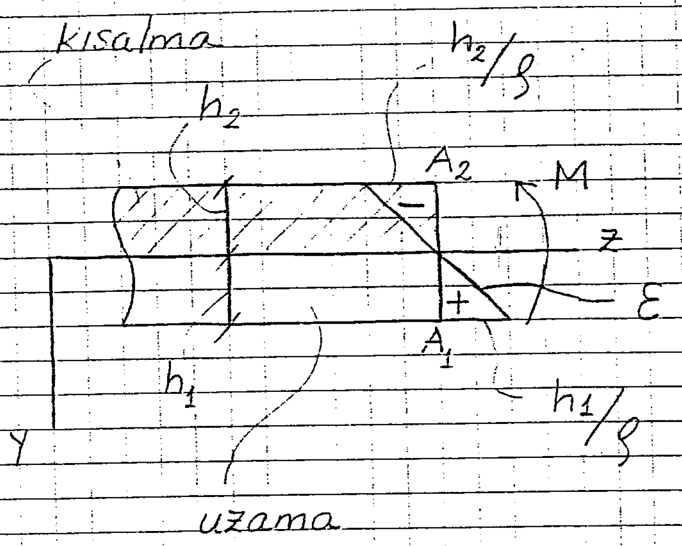
$$\varepsilon = \frac{\overline{B'B'} - \overline{BB}}{\overline{BB}}$$

$$= \frac{(\rho + y)\Delta\phi - \underbrace{\Delta z}_{\rho\Delta\phi}}{\underbrace{\Delta z}_{\rho\Delta\phi}}$$

$$= \frac{y * \Delta\phi}{\rho * \Delta\phi}$$

$$\Rightarrow \boxed{\varepsilon = \frac{y}{\rho}}$$

TD'den y-mesafedeki lifciğin birim uzaması



$$\varepsilon = \frac{y}{\varrho}$$

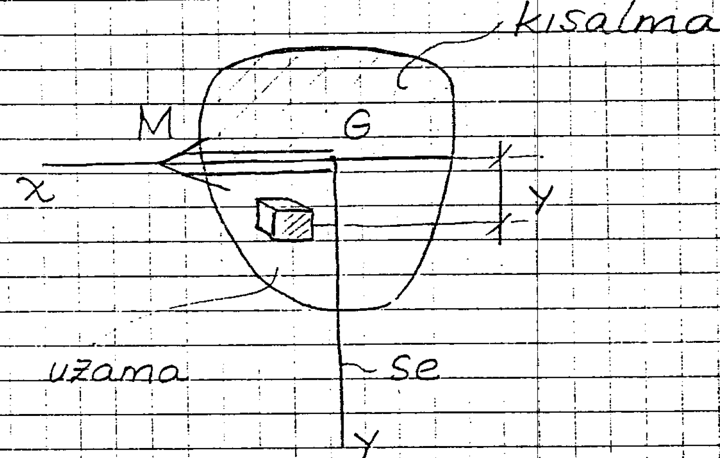
$\rightarrow A_1 \Rightarrow y = h_1 \Rightarrow \varepsilon = \dfrac{h_1}{\varrho}$

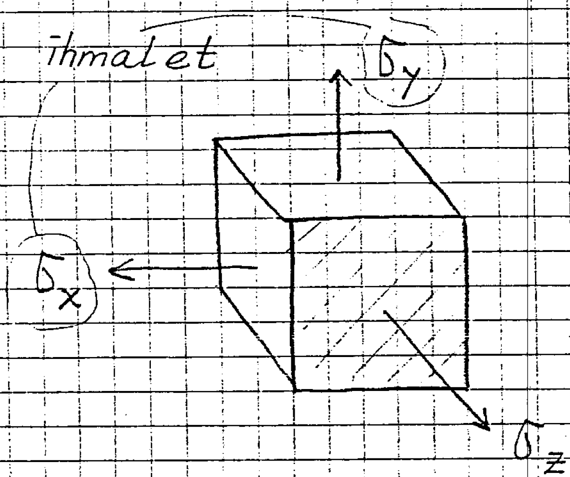
$\rightarrow y = 0 \Rightarrow \varepsilon = 0$

$\rightarrow A_2 \Rightarrow y = -h_2 \Rightarrow \varepsilon = -\dfrac{h_2}{\varrho}$

"y" ye göre
değişim
doğrusal

GERİLME İFADESİ





Bünye Denklemi :

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}\left(\cancelto{0}{\sigma_x} + \cancelto{0}{\sigma_y}\right)$$

σ

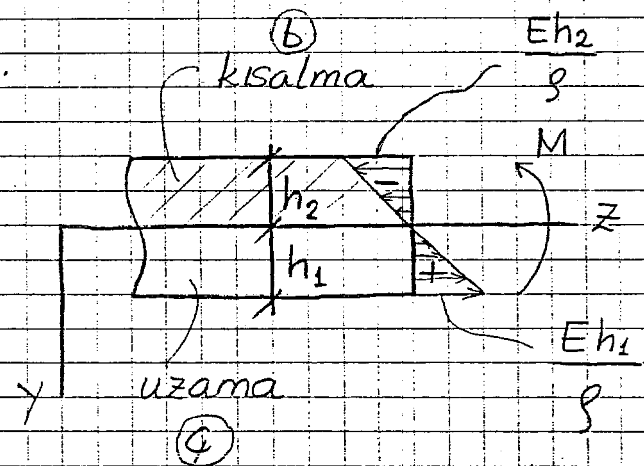
$$\Rightarrow \sigma_z = E\,\varepsilon_z$$

ε

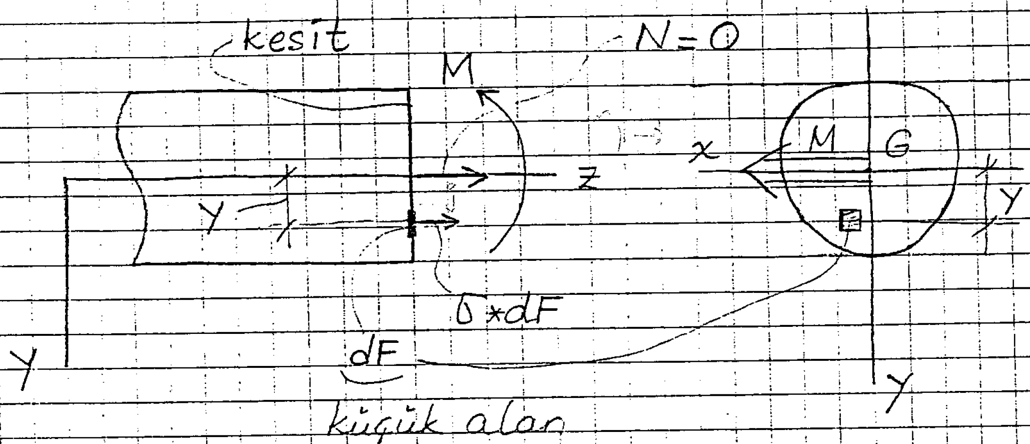
$$\Rightarrow \quad \sigma = E\varepsilon \sim \frac{y}{\rho}$$

$$\Rightarrow \quad \boxed{\sigma = \frac{Ey}{\rho}}$$

"y" ye doğrusal bağlı



σ – M BAĞINTISI



① $M = \int_F y \underbrace{\sigma}_{\frac{Ey}{\rho}} dF$

② $N = \int_F \underbrace{\sigma}_{\frac{E}{\rho}y} dF = 0$? sağlanıyor

①:

$$\Rightarrow M = \int_F y * \overbrace{\frac{Ey}{\rho}}^{\sigma} dF$$

$$= \frac{E}{\rho} \underbrace{\int y^2 dF}_{I_x}$$

$$\Rightarrow \boxed{M = \frac{EI_x}{\rho}}$$

$$⊛ \Rightarrow \boxed{\frac{1}{\rho} = \frac{M}{EI_x}}$$ Eğrilik – moment bağıntısı

$EI_x \rightarrow$ eğilme rijitliği

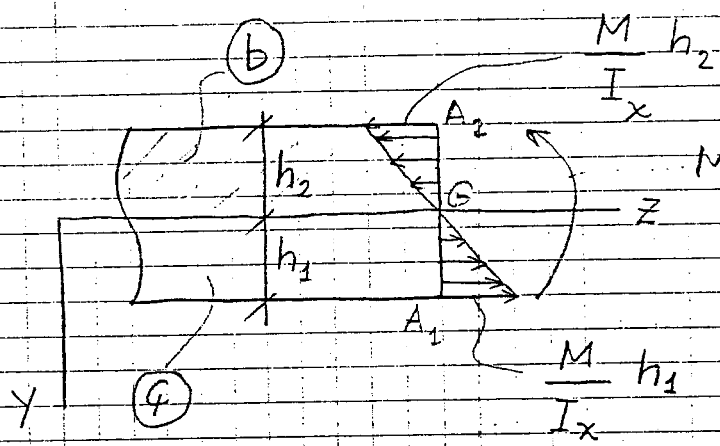
2 :

$$\Rightarrow N = \frac{E}{\rho} \underbrace{\int_F y\,dF}_{0} = 0$$

$$\sigma = \frac{Ey}{\rho} = Ey \underbrace{\left(\frac{1}{\rho}\right)}_{\frac{M}{EI_x}}$$

$$\Rightarrow \sigma = Ey * \frac{M}{EI_x}$$

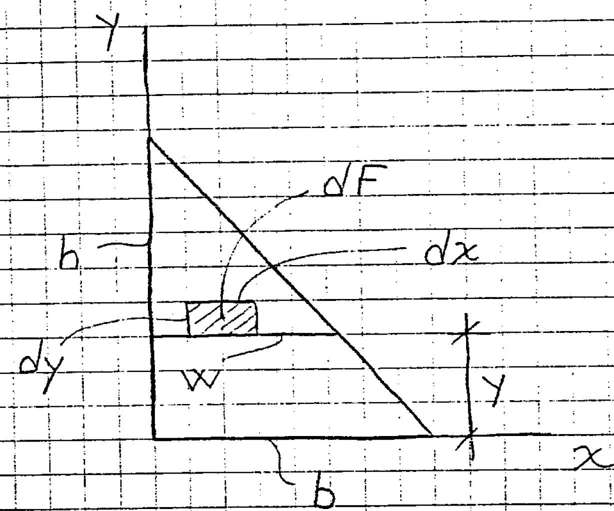
$$\Rightarrow \boxed{\sigma = \frac{M}{I_x} y}$$ $\sigma - M$ bağıntısı



$$\sigma = \frac{M}{I_x} y \begin{cases} A_2 \Rightarrow y = -h_2 \\ \Rightarrow \underbrace{\sigma = -\frac{M}{I_x} h_2}_{\sigma_2 \rightarrow \text{en büyük b gerilmesi}} \\ G \Rightarrow y = 0 \Rightarrow \sigma = 0 \\ A_1 \Rightarrow y = h_1 \\ \Rightarrow \underbrace{\sigma = \frac{M}{I_x} h_1}_{\sigma_1} \end{cases}$$

max ç gerilmesi

Atalet momenti için ek örnek:



$$I_{xy} = ? \neq 0$$

$$\frac{w}{b} = \frac{h-y}{h}$$

$$\Rightarrow \quad w = \frac{b}{h}(h-y)$$

$$I_{xy} = \int_F xy \, \underbrace{dF}_{dx\,dy}$$

$$\Rightarrow I_{xy} = \iint xy \, dx\,dy$$

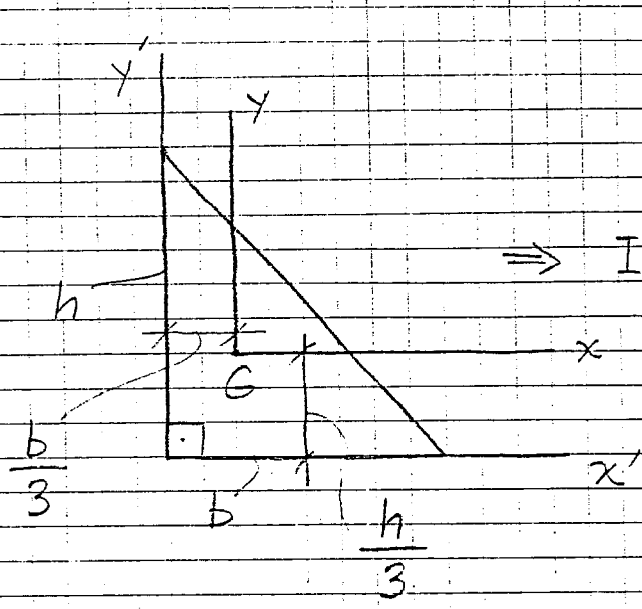
$$= \int_0^h y \left[\int_0^{w} x\,dx \right] dy, \qquad w = \frac{b}{h}(h-y)$$

$$= \int_0^h y \underbrace{\left[\frac{x^2}{2}\right]_0^{\frac{b}{h}(h-y)}}_{\frac{1}{2}\frac{b^2}{h^2}(h-y)^2 - 0} dy$$

$$\Rightarrow \quad I_{xy} = \int_0^h y \frac{b^2}{2h^2} \overbrace{(h-y)^2}^{h^2-2hy+y^2} dy$$

$$= \frac{b^2}{2h^2} \underbrace{\int_0^h \left[h^2 y - 2hy^2 + y^3\right] dy}_{\underbrace{\left[\frac{h^2y^2}{2} - \frac{2hy^3}{3} + \frac{y^4}{4}\right]_0^h}_{\frac{h^4}{12}}}$$

$$\Rightarrow \quad \boxed{I_{xy} = \frac{b^2h^2}{24}}$$



$$\Rightarrow I_{xy} = ?$$

$$\underbrace{I_{x'y'}}_{\frac{b^2h^2}{24}} = I_{xy} + \underbrace{\left(+\frac{b}{3}\right)\left(+\frac{h}{3}\right) * \frac{bh}{2}}_{\frac{b^2h^2}{18}}$$

$$\Rightarrow \boxed{I_{xy} = -\frac{b^2h^2}{72}}$$

$$\underbrace{\sigma_{max}}_{\text{mutlak max gerilme}} = max\left(|\sigma_1|, |\sigma_2|\right)$$

$$max\left(h_1, h_2\right) = h_{max}$$

$$\Rightarrow \boxed{\sigma_{max} = \frac{M}{I_x} h_{max}}$$

$$= \frac{M}{\underbrace{\left(I_x / h_{max}\right)}_{W_x}}$$

$W_x \rightarrow$ kesitin x-etrafındaki mukavemet momenti

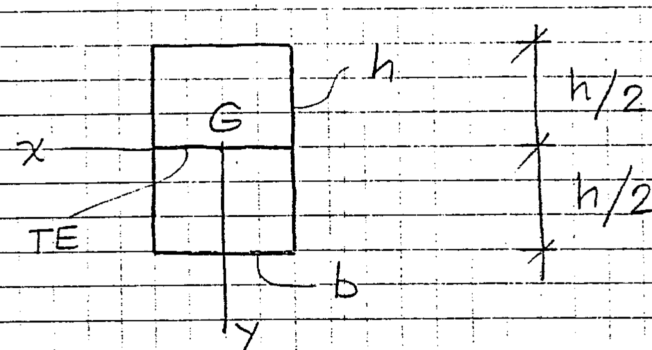
$$\left[I_x\right] = L^4$$

$$\left[W_x\right] = L^3 \rightarrow cm^3, m^3 \text{ -- vs.}$$

$$\Rightarrow \boxed{\begin{array}{l} \sigma_{max} = \dfrac{M}{W_x} \\ W_x = \dfrac{I_x}{h_{max}} \\ h_{max} = max\left(h_1, h_2\right) \end{array}}$$

Mukavemet momenti ne kadar büyük olursa kesitte oluşacak σ_{max} o kadar küçük olur ve dolayısıyla kesitin eğilmeye karşı dayanımı iyileşir.

Dikdörtgen kesit için W_x



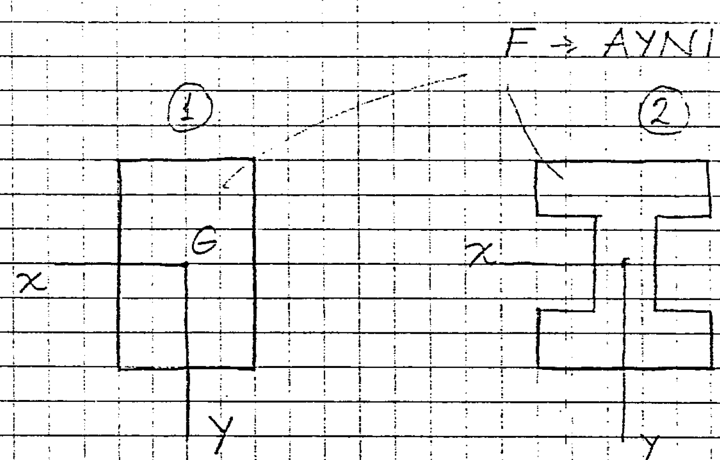
$$I_x = \frac{bh^3}{12}$$

$$h_{max} = \frac{h}{2} \Rightarrow W_x = \frac{bh^{\cancel{3}\,2} / \cancel{12}\,6}{\cancel{h} / \cancel{2}}$$

$$\Rightarrow \boxed{W_x = \frac{bh^2}{6}}$$

Kural:

Kesit malzemesi ne kadar çok dışa doğru yığılırsa kesitin mukavemet momenti o kadar büyük olur ve dolayısıyla kesitin eğilmeye karşı mukavemeti artar.

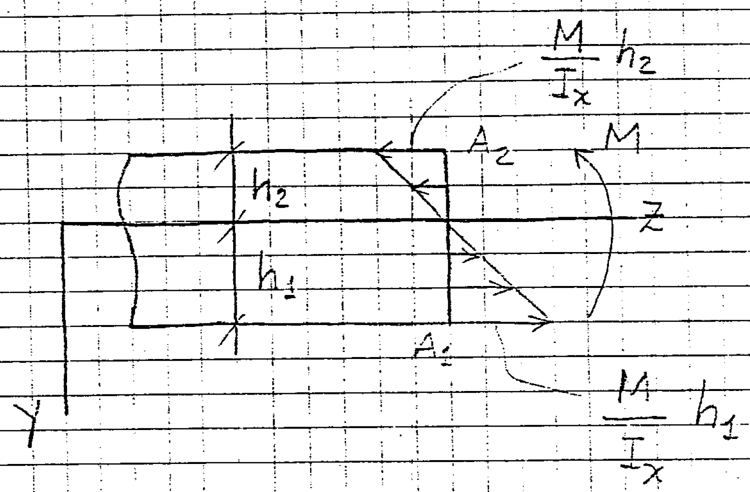


$W_x^{(1)}$ $\qquad$ $W_x^{(2)}$

$$W_x^{(1)} < W_x^{(2)}$$

(2) kesitinin eğilmeye karşı dayanımı daha iyidir.

BOYUTLANDIRMA



$$\sigma = \frac{M}{I_x} y$$

$$A_1 \rightarrow \sigma_1 = \frac{M}{I_x} h_1$$

$$A_2 \rightarrow \sigma_2 = -\frac{M}{I_x} h_2$$

$$\sigma_{max} = \frac{M}{W_x} \qquad W_x = \frac{I_x}{h_{max}}$$

>0

$\sigma_{em} \rightarrow$ (ç) emniyet gerilmesi

$\sigma'_{em} \rightarrow$ (b) emniyet gerilmesi

$$\sigma_1 = \frac{M}{I_x} h_1 \leq \sigma_{em}$$

$$|\sigma_2| = \frac{M}{I_x} h_2 \leq \sigma'_{em}$$

Genel boyutlandırma denklemleri

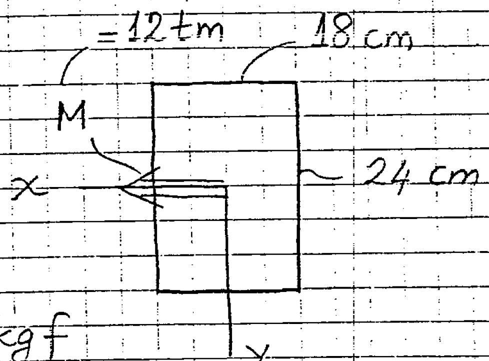
BD

Eğer $\sigma_{em} = \sigma'_{em}$ ise BD şu şekilde yazılabilir:

$$\boxed{\sigma_{max} = \frac{M}{W_x} \leq \sigma_{em}}$$

$\sigma_{em} = \sigma'_{em}$ ise geçerli

<u>**ÖRNEK:**</u>



$\sigma_{em} = \sigma'_{em}$

$= 700 \dfrac{kgf}{cm^2}$

$\Rightarrow$ kesit yeterli mi?

$$\sigma_{max} = \frac{M}{W_x} \overset{?}{\leq} \sigma_{em}$$ 700

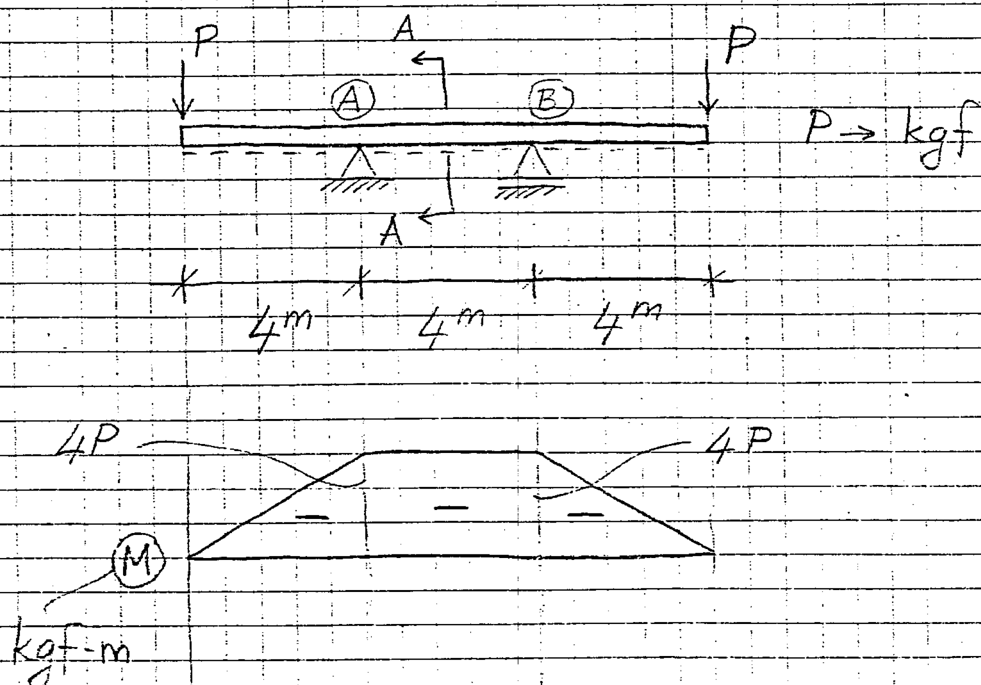
$$W_x = \frac{bh^2}{6} = \frac{(18)*(24)^2}{6} = 1728 \ cm^3$$

$$M = 12 \ tm = 12*10^5 \ kgf\text{-}cm$$

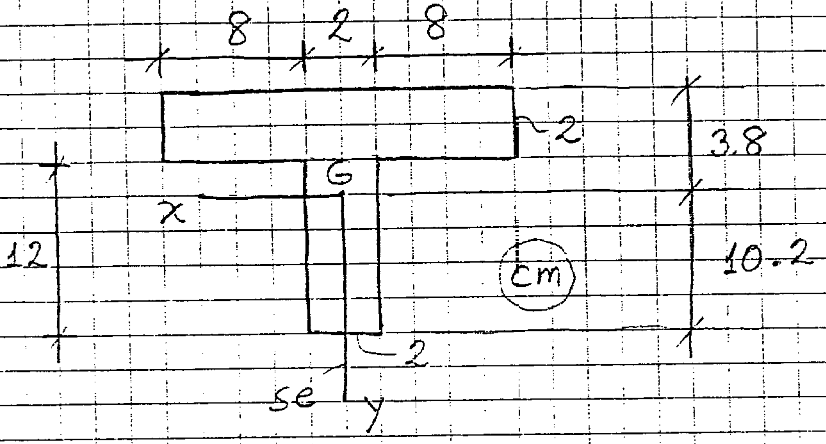
$$\Rightarrow \sigma_{max} = \frac{12\times10^5}{1728} = 694 \leq 700$$

$\Rightarrow$ KESİT YETERLİ

ÖRNEK:



A-A :



$$\sigma_{em} = \sigma'_{em} = 1200 \ kgf/cm^2$$

Kesitin emniyetle taşıyabileceği max P-kuvveti soruluyor.

$$I_x = \frac{18 \times 2^3}{12} + (-2{,}8)^2 \times 36 + \frac{2 \times 12^3}{12} + (4{,}2)^2 \times 24$$

kontrol ok

$$= 1006 \ cm^4$$

$$\sigma_{max} = \frac{M}{W_x} \leq \underbrace{\sigma_{em}}_{1200}$$

$|M|_{max} = 4P \ kgf-m$

$400 P \ kgf-cm$

$$W_x = \frac{\overset{1006}{I_x}}{\underset{10.2}{h_{max}}} \simeq 97 \ cm^3$$

kuvvet → kgf

uzunluk → cm

$$\Rightarrow \frac{400 P}{97} \leq 1200$$

$$\Rightarrow P \leq 291 \text{ kgf}$$

$$\Rightarrow \boxed{P_{max} = 291 \text{ kgf}}$$

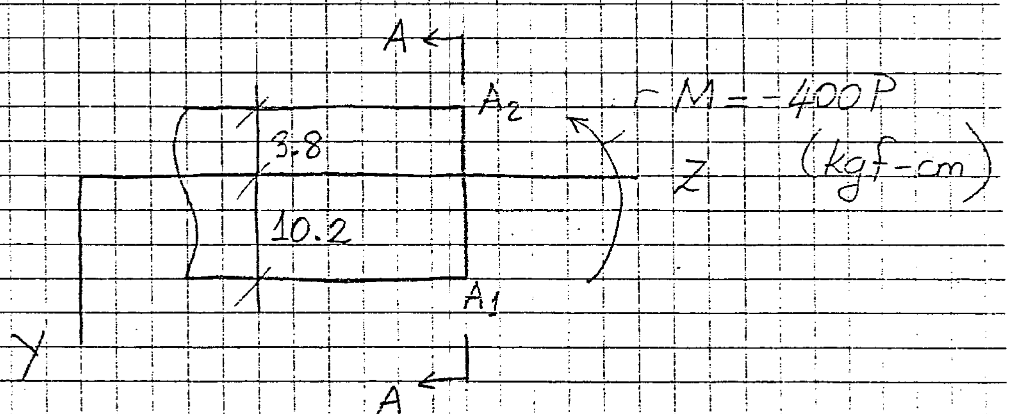
ÖRNEK:

Bir önceki örnek

$$\sigma_{em} = 1200 \text{ kgf}/cm^2$$

$$\sigma'_{em} = 800 \text{ kgf}/cm^2$$

$$\Rightarrow P_{max} = ?$$



$$\sigma = \frac{M}{I_x} y$$

$A_1 \rightarrow y = 10.2 \text{ cm}$

(b)

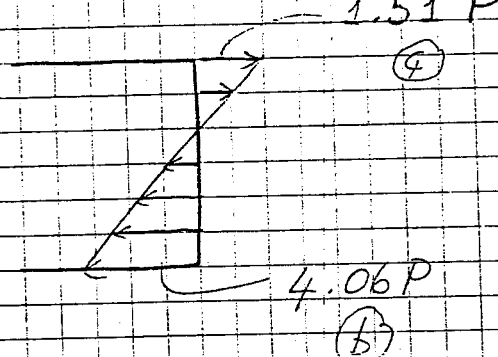
$$\Rightarrow \sigma = \frac{-400P}{1006} * (10.2) \cong \underbrace{-4.06P}_{}$$

kgf/cm^2

$A_2 \rightarrow y = -3.8 \text{ cm}$

(a)

$$\Rightarrow \sigma = \frac{-400P}{1006} * (-3.8) \cong 1.51P$$



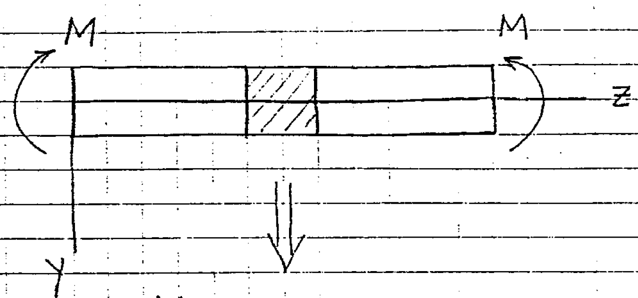
$$\Rightarrow \underbrace{|-4.06P|}_{4.06P} \leq \underbrace{\sigma'_{em}}_{800}$$

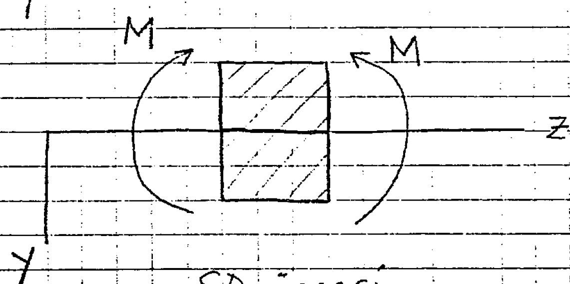
$$\Rightarrow P \leq 197 \text{ kgf}$$

$$1.51P \leq \underbrace{\sigma_{em}}_{1200} \Rightarrow P \leq 795 \text{ kgf}$$

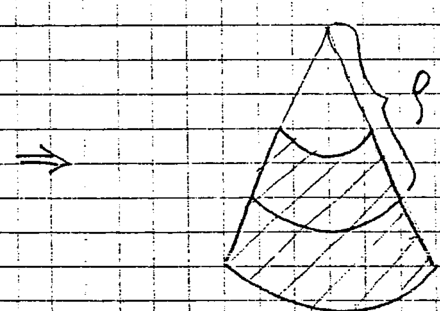
$$\Rightarrow \boxed{P_{max} = 197 \text{ kgf}}$$

Hatırla:





ŞD öncesi



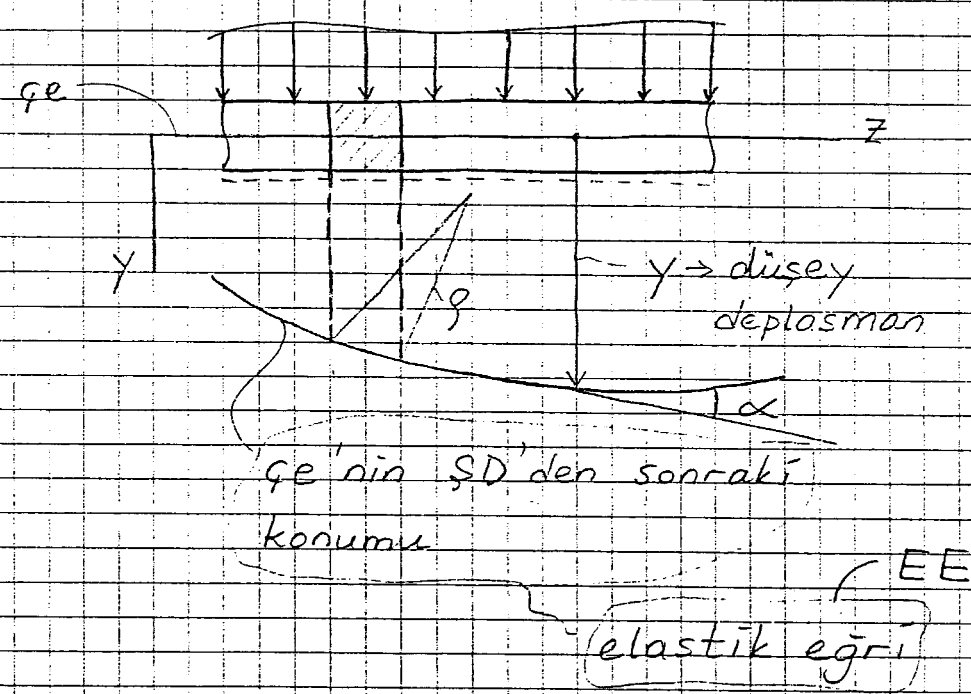
ŞD sonrası

$$\boxed{\frac{1}{\rho} = \frac{M}{EI_x}}$$

eğrilik-moment bağıntısı

eğrilik

Küçük ŞD hali için eğrilik-moment bağıntısı



$$\rho = \rho(z)$$

$$y = y(z)$$ EE'yi tanımlayan denklem

$$\tan\alpha = y' = \frac{dy}{dz}$$ → EE nin eğimi

Eğrilik-moment bağıntısı:

$$\frac{1}{\rho} = \frac{M}{EI_x} \quad (*)$$

$$\frac{1}{\rho} = \frac{\mp y''}{\left(1+(y')^2\right)^{3/2}} \quad (A) \qquad y' = \frac{dy}{dz}$$

$$y'' = \frac{d^2y}{dz^2}$$

küçük ŞD hali için:

$$|y'| << 1$$

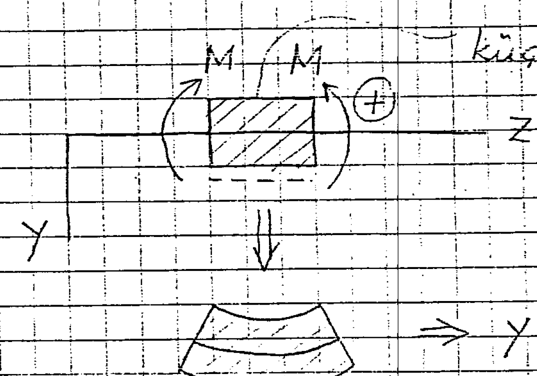
$$\Rightarrow (A): \quad \boxed{\frac{1}{\rho} = \mp y''}$$

küçük ŞD için geçerli

$$\Rightarrow (*): \quad \boxed{y'' = \mp \frac{M}{EI_x} \quad (**)}$$

<u>$\mp$ nin belirlenmesi</u>

küçük eleman



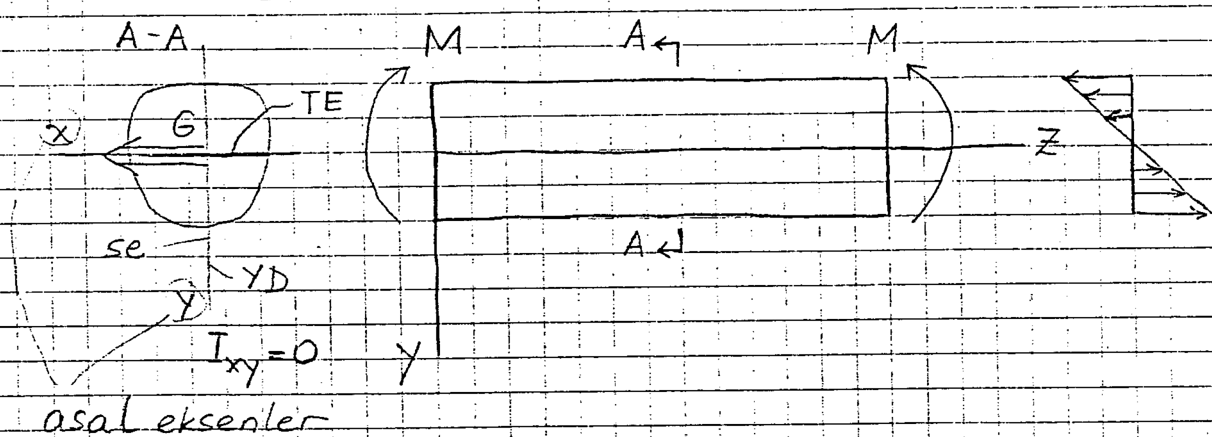
$\Rightarrow y'' < 0$

$\Rightarrow$ M (+) ise $\rightarrow$ y'' (−)

$$\Rightarrow \quad (**) : \quad \boxed{y'' = -\frac{M}{EI_x}}$$

küçük ŞD hali için
eğrilik-moment bağıntısı

Hatırla:

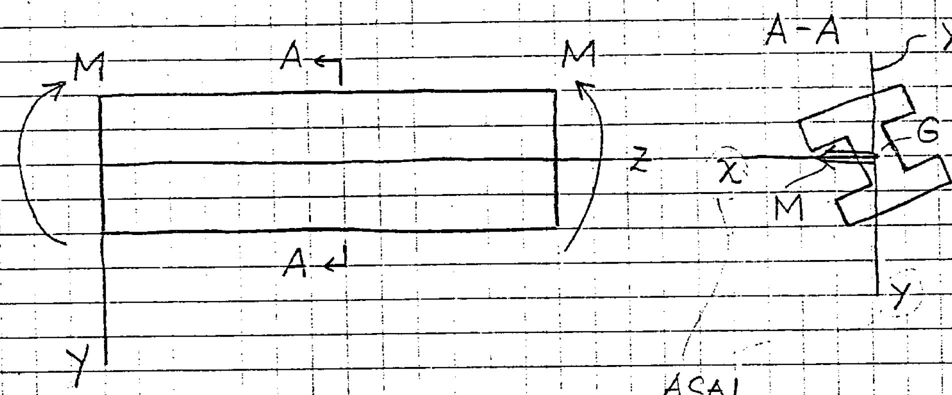


YD
Yükleme düzlemi:
yz (düşey) düzlemi

$\Rightarrow$ Kesitin eğilmesi $\rightarrow$ TE etrafında

$\Rightarrow$ eğilme yz (YD) düzleminde

EĞİK EĞİLME



$I_{xy} \neq 0$

YD → yz düzlemi

M'nin döndürme etkisi YD'de

↓

Çubuğun eğilmesi YD'de olmayacak.

x-ekseni TE olmayacak

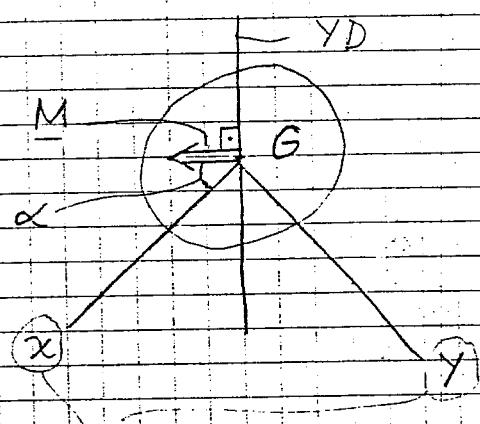
= YD

Kuvvetler düzlemi kesitin asal eksenlerinden birisi ile çakışmazsa eğilme eğik olur. Yani eğilme kuvvetler düzleminde olmaz.

Eğik eğilme problemi süperpozisyon kuralı kullanılarak çözülebilir.

Basamak 1: Kesite etki eden moment bulunur

Basamak 2: Kesitin asal eksenleri bulunur.



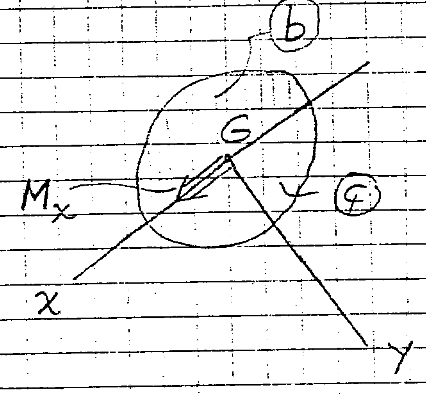
ASAL EKSENLER $\Rightarrow I_{xy} = 0$

$$\left.\begin{aligned} M_x &= M\cos\alpha \\ M_y &= -M\sin\alpha \end{aligned}\right)$$ M'nin x, y bileşenleri.

$$\underline{M} = (M_x, M_y)$$

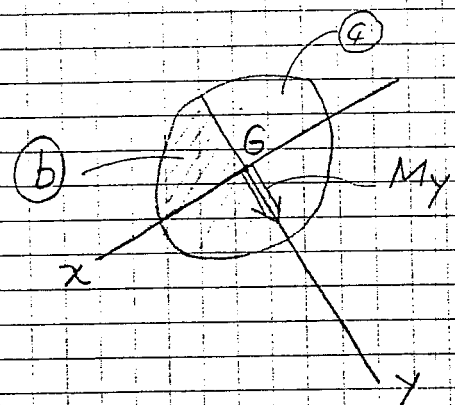
Basamak 3: x ve y asal eksenleri etrafındaki eğilme problemleri çözülür.

x-etrafındaki eğilme problemi:



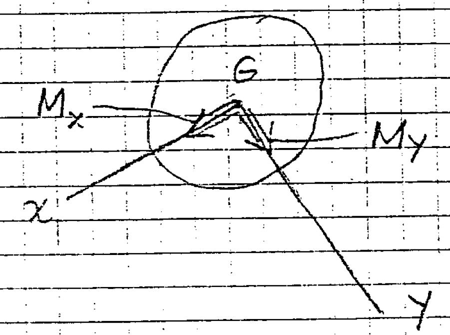
$$\boxed{\sigma = \frac{M_x}{I_x}\, y}$$

y-etrafındaki eğilme problemi:



$$\boxed{\sigma = -\frac{M_y}{I_y}\, x}$$

Basamak 4: Toplam gerilmeyi bulmak için Basamak 3'de elde edilen sonuçlar toplanır.



$$\underbrace{\boxed{\sigma = \frac{M_x}{I_x}\, y - \frac{M_y}{I_y}\, x}}_{\text{toplam gerilme}}$$

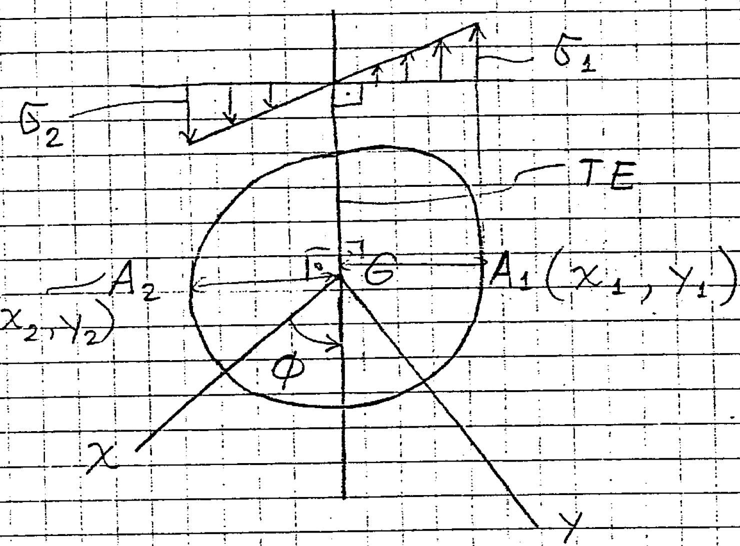
TE'nin yeri:

TE de $\sigma = 0$

$$\Rightarrow \sigma = \boxed{\frac{M_x}{I_x} y - \frac{M_y}{I_y} x = 0}$$

$$\Rightarrow y = \underbrace{\left(\frac{M_y}{M_x} \frac{I_x}{I_y} \right)}_{Tan\phi} x$$

xy-eksen takımında TE'nin denklemi — orijinden geçen, eğimi $Tan\phi$ olan DOĞRU



$\phi \rightarrow x$ ve TE arasındaki açı

$\phi \rightarrow (+) \Rightarrow (x)$ den (y) ye doğru

$\phi \rightarrow (-) \Rightarrow$ ters yönde

$$\boxed{\tan \phi = \frac{M_y}{M_x}\,\frac{I_x}{I_y}}$$

Eğilme TE etrafında ve maximum gerilmeler TE'e en uzak noktalarda olacak.

TE'de $\sigma = 0$

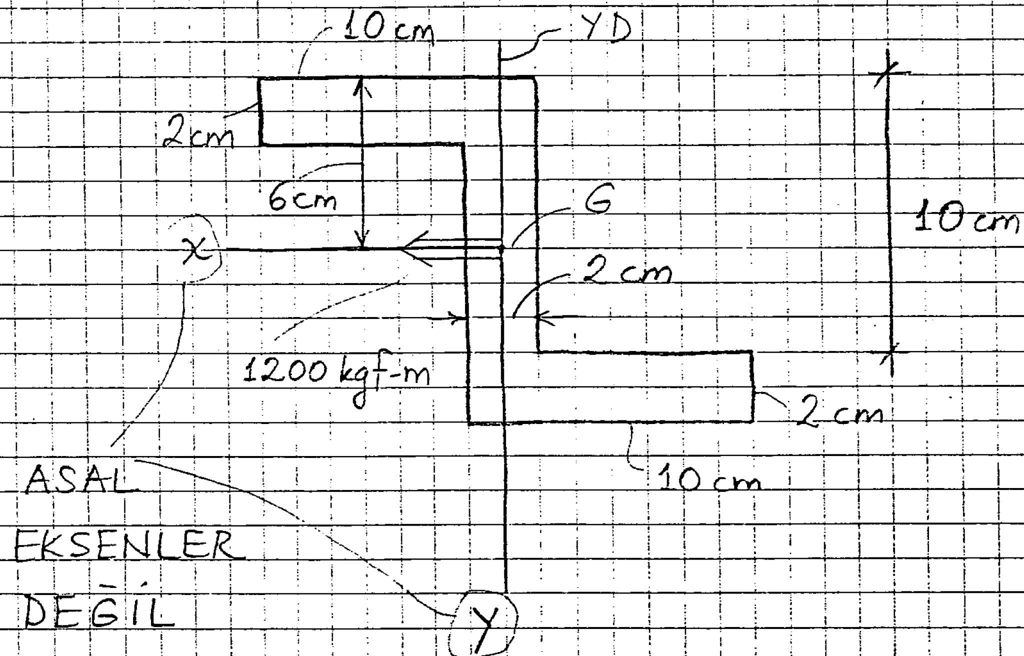
$\left.\begin{matrix}\sigma_1\\ \sigma_2\end{matrix}\right)$ max gerilmeler

$$\sigma = \frac{M_x}{I_x}\,y - \frac{M_y}{I_y}\,x$$

$$\Rightarrow \quad \sigma_1 = \sigma\Big|_{A_1} = \frac{M_x}{I_x}\,y_1 - \frac{M_y}{I_y}\,x_1$$

$$\sigma_2 = \sigma\Big|_{A_2} = \frac{M_x}{I_x}\,y_2 - \frac{M_y}{I_y}\,x_2$$

ÖRNEK:

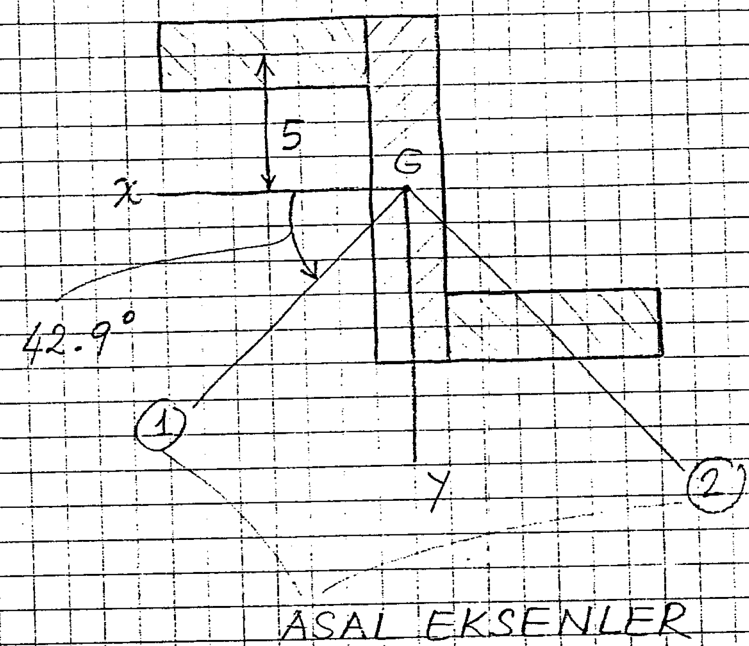


$\Rightarrow \quad I_{xy} \neq 0$

Şekildeki kesit $M = 1200$ kgf-m'lik eğilme momentine maruzdur. Gerilme dağılımını TE'den dik uzaklığa göre belirleyiniz.

$\Rightarrow$ Eğik eğilme

$\Rightarrow$ Asal eksenler ve asal atalet momentleri ?



$$I_x = \frac{2*12^3}{12} + \left[\frac{8*2^3}{12} + (-5)^2 * 16\right] * 2$$

$$\Rightarrow \boxed{I_x \simeq 1099\ cm^4}$$

$$\Rightarrow \boxed{I_y \simeq 979\ cm^4}$$

$$I_{xy} = 0 + \left[0 + (+5)(-5) * 16\right] + \left[0 + (-5)(+5) * 16\right]$$

$$\Rightarrow \boxed{I_{xy} = -800\ cm^4}$$

$$r = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2} \cong 802\ cm^4$$

$$\tan 2\alpha = -\frac{2 I_{xy}}{(I_x - I_y)} = 13.33$$

$$\Rightarrow 2\alpha = 85.71^\circ \cancel{+\pi} \ ?$$

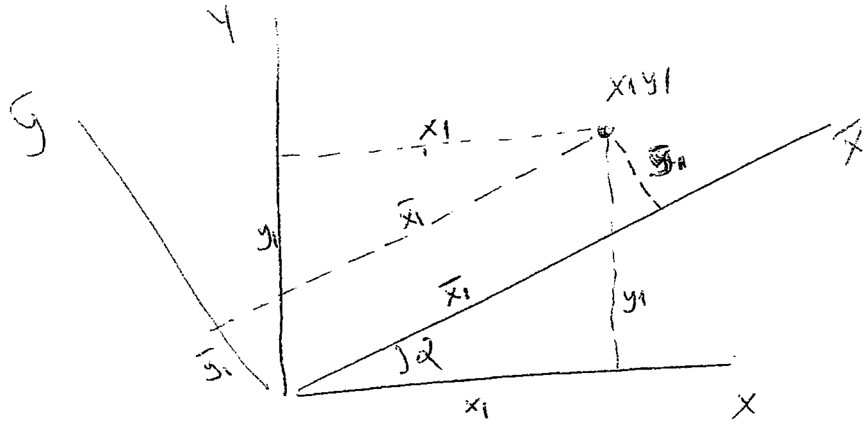
$$\sin 2\alpha = -\frac{I_{xy}}{r} > 0$$

$$\Rightarrow \boxed{\alpha \cong 42.9^\circ}$$

$$I_{12} = 0$$

$$I_1 = \frac{I_x + I_y}{2} + r \cong 1841\ cm^4$$

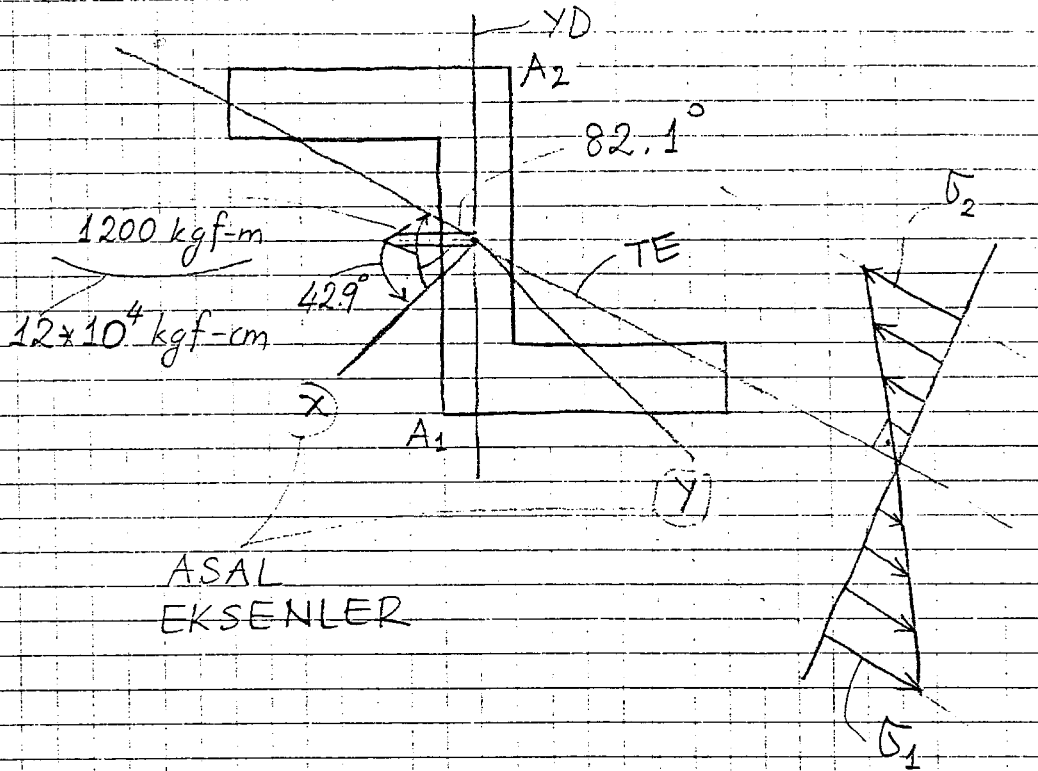
$$I_2 = \frac{I_x + I_y}{2} - r \cong 237\ cm^4$$



$$\begin{bmatrix} \bar{X}_1 \\ \bar{y}_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} X_1 \\ y_1 \end{bmatrix}$$

$$\bar{X}_1 = X_1 \cos\alpha + y_1 \sin\alpha$$
$$\bar{y}_1 = -X_1 \sin\alpha + y_1 \cos\alpha$$

Dönüşüm formülü



$\Rightarrow \quad I_x \simeq 1841 \ cm^4$

$I_y \simeq 237 \ cm^4$

ASAL ATALET MOMENTLERİ

TE:

$$Tan\,\phi = \frac{M_y}{M_x} * \frac{I_x}{I_y}$$

$$M_x = 12*10^4 * Cos\,42.9° \cong 8.79*10^4 \ kgf\text{-}cm$$

$$M_y = -12*10^4 * Sin\,42.9° \cong -8.17*10^4 \ kgf\text{-}cm$$

$$\Rightarrow Tan\,\phi = -7.22 \Rightarrow \boxed{\phi = -82.10°}$$

$A_1 \to (\underbrace{x_1}_{4.82\ cm}, \overbrace{y_1}^{3.71\ cm})$

$A_2 \to (\underbrace{x_2}_{-4.82}, \overbrace{y_2}^{-3.71})$

$$\sigma = \frac{M_x}{I_x} y - \frac{M_y}{I_y} x$$

$$\sigma_1 = \sigma \Big|_{A_1} = \frac{\overbrace{M_x}^{8.79 \times 10^4}}{\underbrace{I_x}_{1841}} \underbrace{y_1}_{3.71} - \frac{\overbrace{M_y}^{-8.17 \times 10^4}}{\underbrace{I_y}_{237}} \underbrace{x_1}_{4.82}$$

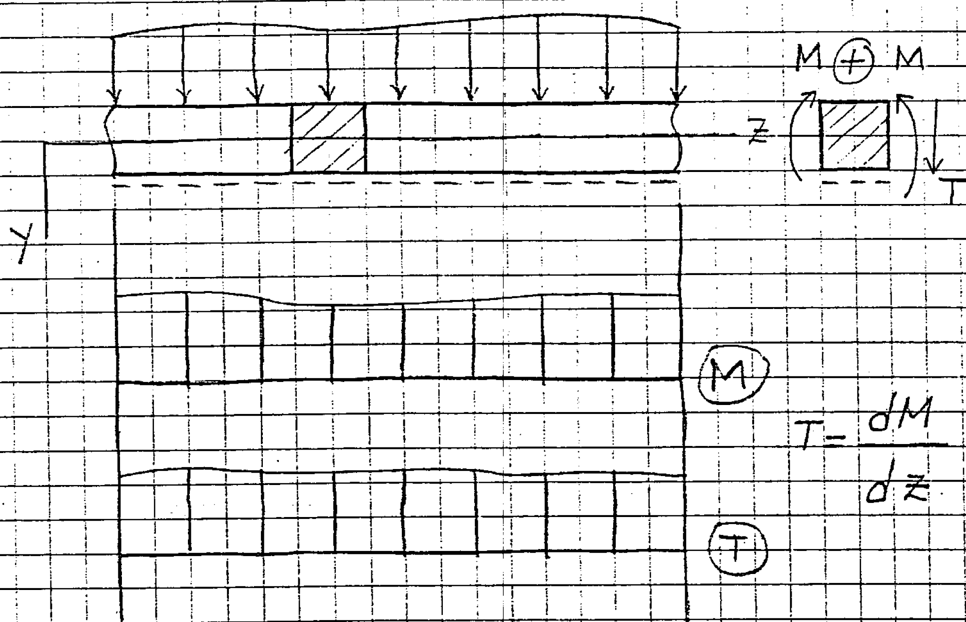
$$\Rightarrow \boxed{\sigma_1 \simeq 1838\ kgf/cm^2 \ c}$$

$$\sigma_2 = \sigma \Big|_{A_2} = \frac{M_x}{I_x} \underbrace{y_2}_{-3.71} - \frac{M_y}{I_y} \underbrace{x_2}_{-4.82}$$

$$\Rightarrow \boxed{\sigma_2 \simeq -1838\ kgf/cm^2 \ b}$$

BÖLÜM 3

KESMELİ EĞİLME



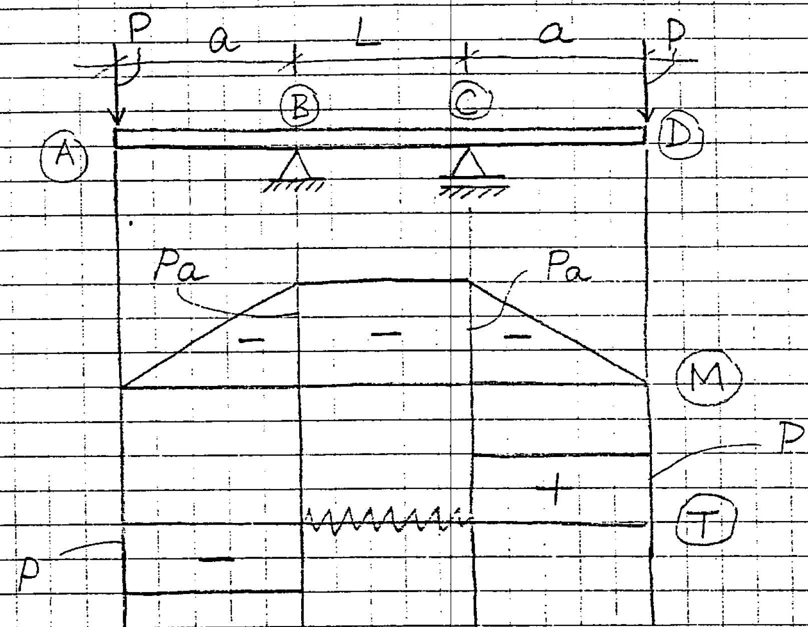
$$T = \frac{dM}{dz}$$

Genel halde:

$$M \neq 0, \quad T \neq 0$$

Hem kesme kuvveti hem de moment etkisi altında meydana gelen çubuk eğilmesine kesmeli eğilme denir.

ÖRNEK :



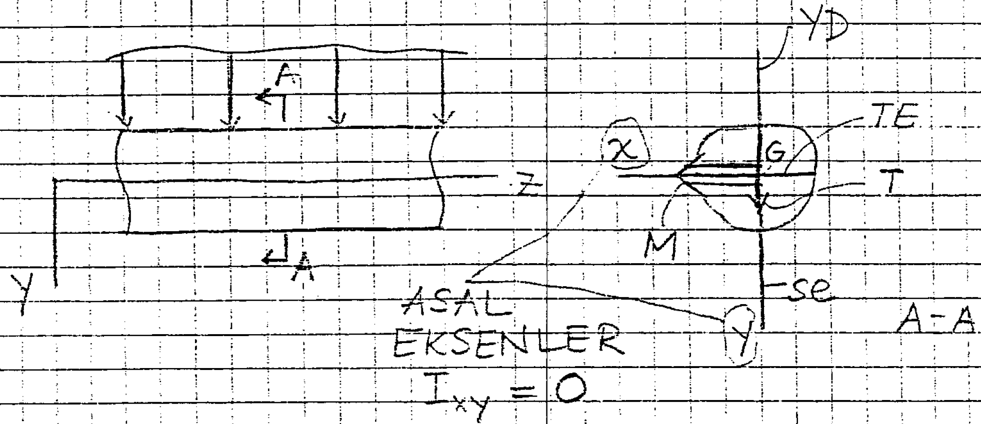
$\Rightarrow$ BC $\rightarrow$ basit eğilmeye maruz

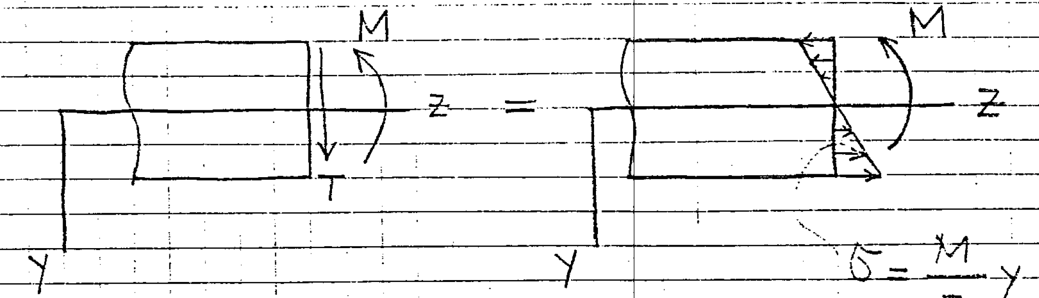
AB, CD $\rightarrow$ kesmeli eğilmeye maruz.

Kabuller :

_ YD $\rightarrow$ düşey düzlem

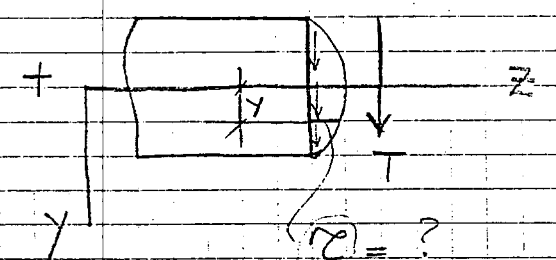
_ Kesit bir simetri eksenine sahip ve bu se YD ile çakışıyor





$$\sigma = \frac{M}{I_x} y$$

BİLİNİYOR

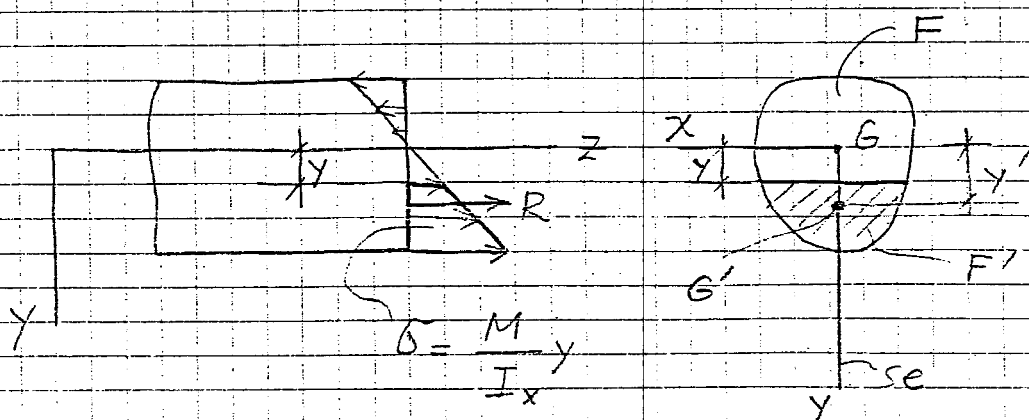


$\tau = ?$

y-mesafesindeki kayma gerilmesi

Amacımız T kesme kuvvetinin meydana getirdiği kayma gerilmelerini bulmaktır.

Hazırlık :



$$\sigma = \frac{M}{I_x} y$$

F : Toplam kesit
G : F'nin ağırlık merkezi
F' : Taralı kısmın alanı
G' : (F')nün ağırlık merkezi
R : (F')ne etki eden normal
gerilmelerin bileşkesi

$R = ?$

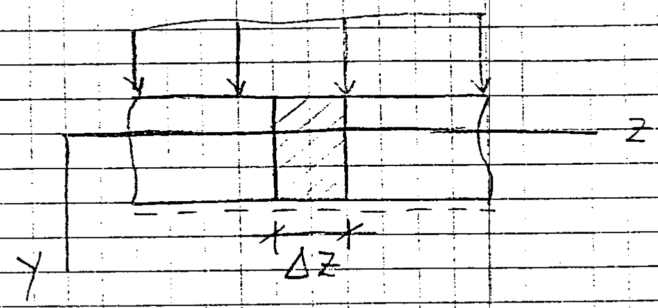
$$R = \int_{F'} \underbrace{\sigma}_{\frac{M}{I_x} y} dF = \frac{M}{I_x} \underbrace{\int y\, dF}_{S_x'}$$

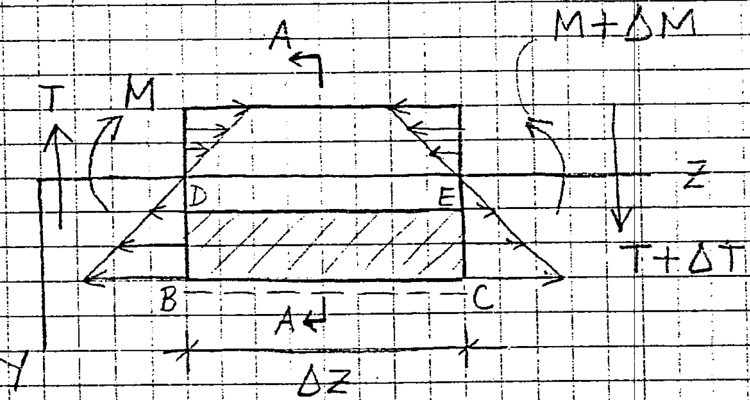
$S_x' \rightarrow$ (F') nün x ekseni etrafındaki statik momenti

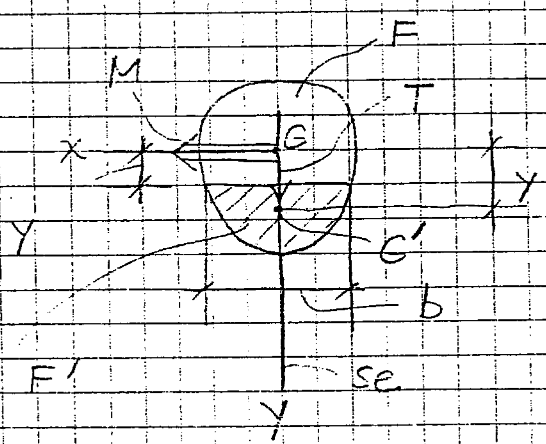
$$\Rightarrow \boxed{R = \frac{M S_x'}{I_x}} \qquad \boxed{S_x' = \int_{F'} y\, dF = y' \cdot F'}$$

$I_x \rightarrow$ Kesitin toplam atalet momenti

Şimdi T kesme kuvvetinin meydana getirdiği kayma gerilmelerini hesaplayalım:

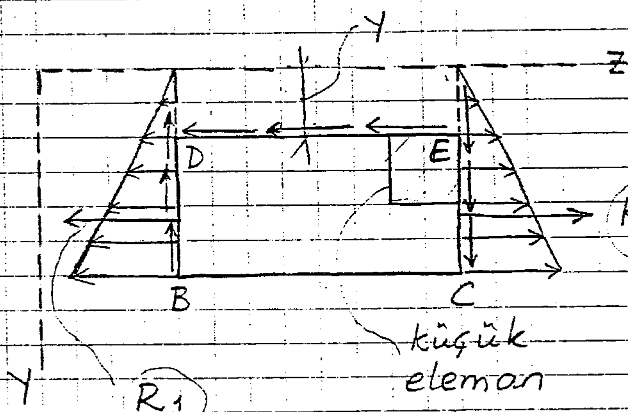






A - A

BCDE için SCD çizelim:



$\frac{(M+\Delta M)S_x'}{I_x}$

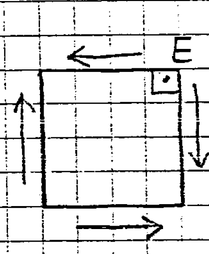
R_2 EC yüzüne etki eden normal gerilmelerin bileşkesi

küçük eleman

R_1 BD yüzüne etki eden normal gerilmelerin bileşkesi

$\frac{M * S_x'}{I_x}$

Küçük elemanın SCD'si:



τ y-mesafesindeki kayma gerilmesi

$\xrightarrow{+}$

$$\sum F_z = 0 \Rightarrow R_2 - R_1 - b\Delta z\,\tau = 0$$

$$\Rightarrow \frac{(M+\Delta M)\,S_x'}{I_x} - \frac{M S_x'}{I_x} - b(\Delta z)\,\tau = 0$$

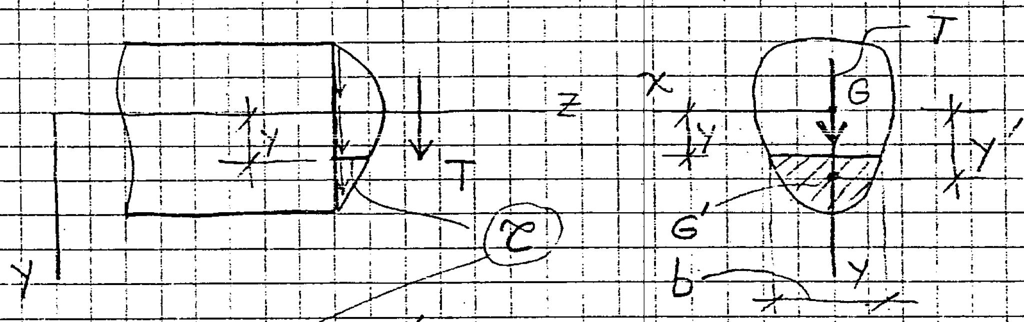
$$\Rightarrow \quad \frac{\Delta M}{I_x} S_x' - b(\Delta z)\, \tau = 0$$

$$\Rightarrow \quad \tau = \frac{S_x'}{b I_x} \frac{\Delta M}{\Delta z}$$

$\Delta z \to 0$ iken:

$$\tau = \frac{S_x'}{b I_x} \underbrace{\frac{dM}{dz}}_{T}$$

$$\Rightarrow \quad \boxed{\tau = \frac{T S_x'}{b I_x}}$$



$$\tau = \frac{T S_x'}{b I_x} \rightarrow \text{y-mesafesindeki kayma gerilmesi}$$

$T \rightarrow$ Kesme Kuvveti

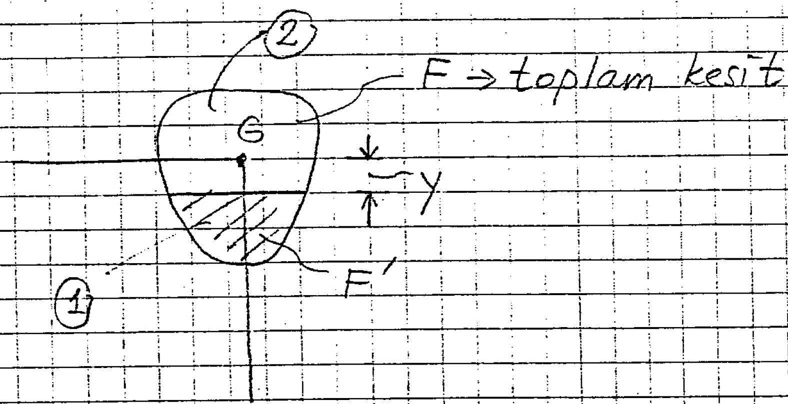
$I_x \rightarrow$ F'nin x-etrafındaki atalet momenti

$S_x' \rightarrow (F')$'nün x etrafındaki statik momenti

$b \rightarrow y$ mesafesindeki kesit genişliği

$$\boxed{S_x' = \int_{F'} y\,dF = y'F'}$$

NOT:



$F = \underbrace{F_1}_{F'} + F_2$

$\left.\begin{matrix} S_x' \\ b \end{matrix}\right)$ y ile değişiyor

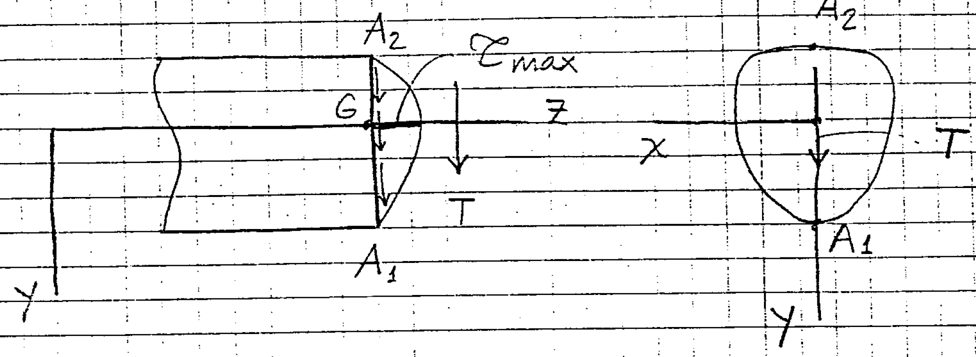
$\underbrace{S_x}_{0} = \underbrace{S_x^{(1)}}_{S_x'} + S_x^{(2)}$

$\left.\begin{matrix} T \\ I_x \end{matrix}\right)$ sabit

$\Rightarrow S_x^{(2)} = -S_x'$

Gözlem: y değiştikçe S_x' ve b değişecek, fakat T ve I_x aynı kalacak

Gözlemler:



$$\tau\Big|_{A_1} = ? \rightarrow S_x' = y'F' = 0$$

$$\Rightarrow \tau\Big|_{A_1} = 0 \text{ olur.}$$

$$\tau\Big|_{A_2} = ? \rightarrow S_x' = \underbrace{y'}_{0}\underbrace{F'}_{F} = 0$$

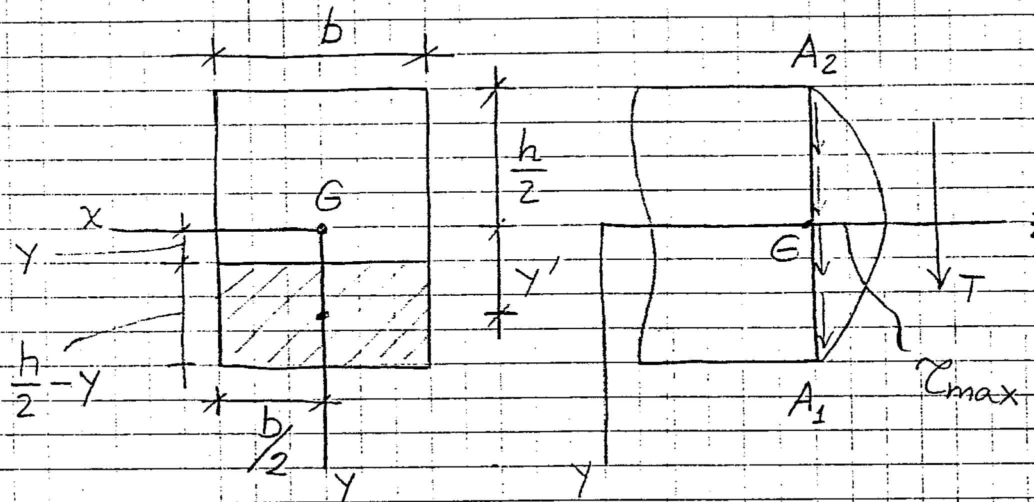
$$\Rightarrow \tau\Big|_{A_2} = 0 \text{ olur}$$

Genel Kural: Kayma gerilmesi max. değerine G'de ulaşır

$$\boxed{\tau_{max} = \tau\Big|_{G} = \tau\Big|_{y=0}}$$

ÖRNEK:

Dikdörtgen kesit için kayma gerilmesi dağılımını bulalım



$$\tau = \frac{T S_x'}{b I_x}$$

$b \cdot b$, $\quad I_x = \frac{bh^3}{12}$

$$S_x' = y' * F'$$

$F' = b\left(\frac{h}{2} - y\right)$

$y' = y + \frac{1}{2}\left(\frac{h}{2} - y\right)$

$$\Rightarrow S_x' = \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)$$

$$\Rightarrow \tau = \frac{T \frac{b}{2}\left(\frac{h^2}{4} - y^2\right)}{b^2 h^3 / 12}$$

$$\Rightarrow \quad \tau = \frac{T}{bh} * \frac{6}{h^2}\left(\frac{h^2}{4} - y^2\right)$$

$$= \frac{T}{bh} * \frac{6}{h^2} * \frac{h^2}{4}\left[1 - \frac{y^2}{(h/2)^2}\right]$$

$$\Rightarrow \quad \tau = \frac{3}{2}\frac{T}{F}\left[1 - \frac{y^2}{(h/2)^2}\right]$$

$$\tau\Big|_{A_1} = \tau\Big|_{y=+h/2} = 0$$

$$\tau\Big|_{A_2} = \tau\Big|_{y=-h/2} = 0$$

$$\tau_{max} = \tau\Big|_{G} = \tau\Big|_{y=0} \Rightarrow \boxed{\tau_{max} = \frac{3}{2}\frac{T}{F}}$$

$$\tau_{max} = \underbrace{\frac{3}{2}}_{k}\overbrace{\frac{T}{F}}^{\tau_{ort}} \Rightarrow \boxed{\tau_{max} = k\,\tau_{ort}}$$

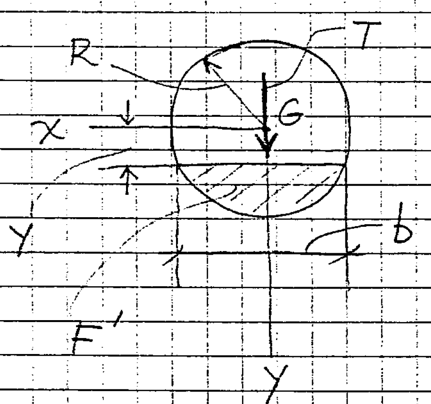
Dikdörtgen kesit için $k = \frac{3}{2}$

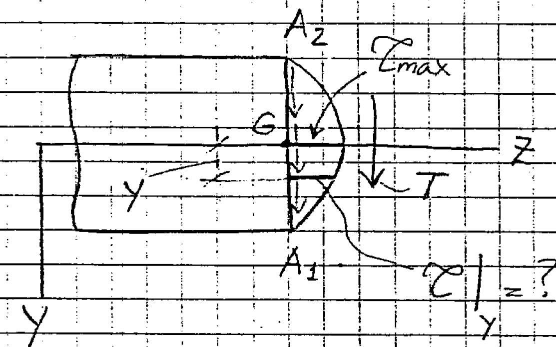
Herzaman $k > 1$ olmalı

k katsayısı kesit şekline göre değişir. k sayısına kayma gerilmesi katsayısı denir.

ÖRNEK:

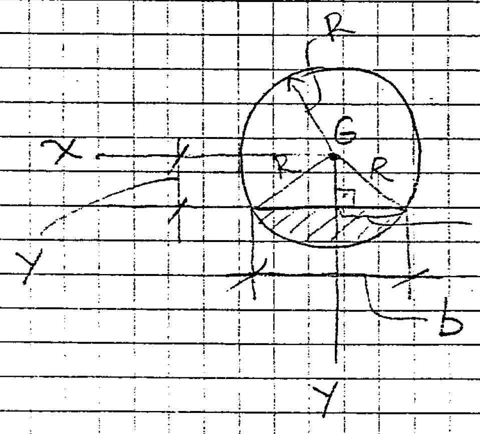
Dairesel kesitte kayma gerilmesi dağılımı





$$\tau\Big|_y = \frac{T S'_x}{b\, I_x} \quad (*)$$

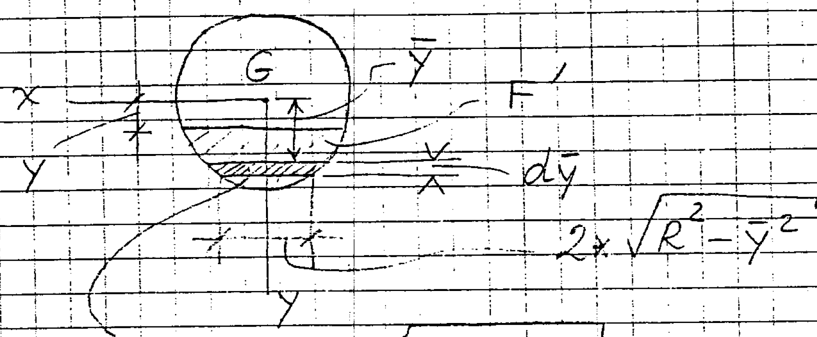
$$I_x = \frac{\pi R^4}{4}$$



$\sqrt{R^2 - y^2}$

$\Rightarrow b = 2 * \sqrt{R^2 - y^2}$ (A)

$$S_x' = \int_{F'} y\, dF$$



$$dF = 2 * \sqrt{R^2 - \bar{y}^2}\; d\bar{y}$$

$$\Rightarrow \quad S_x' = \int_y^R \bar{y} * 2\sqrt{R^2 - \bar{y}^2}\; d\bar{y}$$

$$\Rightarrow \quad \boxed{S_x' = \frac{2}{3}\left(R^2 - y^2\right)^{3/2}} \quad (B)$$

(A), (B) $\rightarrow$ (*) :

$$\Rightarrow \quad \boxed{\tau = \frac{4}{3}\,\frac{T}{\underbrace{F}_{\pi R^2}}\left[1 - \left(\frac{y}{R}\right)^2\right]}$$

$$\tau\Big|_{A_1} = \tau\Big|_{y=R} = 0$$

$$\tau\Big|_{A_2} = \tau\Big|_{y=-R} = 0$$

$$\tau_{max} = \tau\Big|_{G} = \tau\Big|_{y=0}$$

$$\Rightarrow \boxed{\tau_{max} = \underbrace{\frac{4}{3}}_{k}\,\overbrace{\frac{T}{F}}^{\tau_{ort}}}$$

$$\frac{T}{F}$$

$$\Rightarrow \boxed{\tau_{max} = k\,\tau_{ort}}$$

$k = \dfrac{4}{3} \rightarrow$ dairesel kesit için

$k = \dfrac{3}{2} \rightarrow$ dikdörtgen kesit için

herzaman için $k > 1$

$$\tau_{max} = \underbrace{k}_{\text{kayma gerilmesi katsayısı}}\,\overbrace{\frac{T}{F}}^{\tau_{ort}}$$

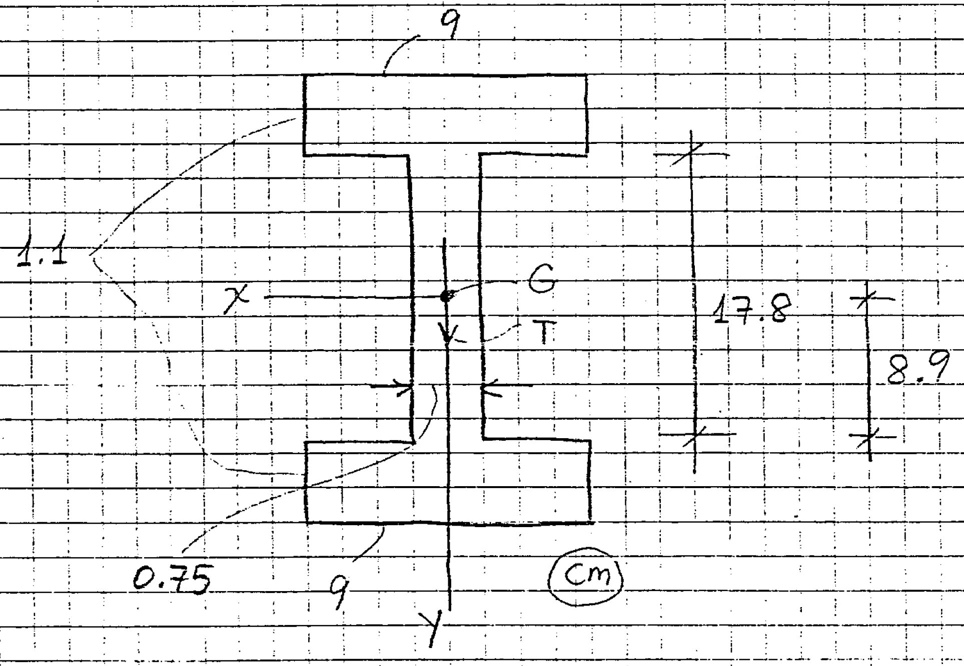
kayma gerilmesi katsayısı

Çeşitli kesitler için "k" katsayıları

kare — eşkenar üçgen — ince halka kesit

Kesit					
k	$\frac{4}{3}$	$\frac{3}{2}$	1.59	1.73	2

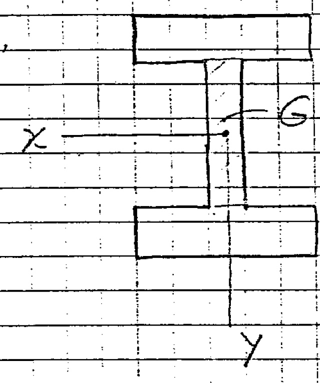
ÖRNEK:



$T = 1000$ kgf

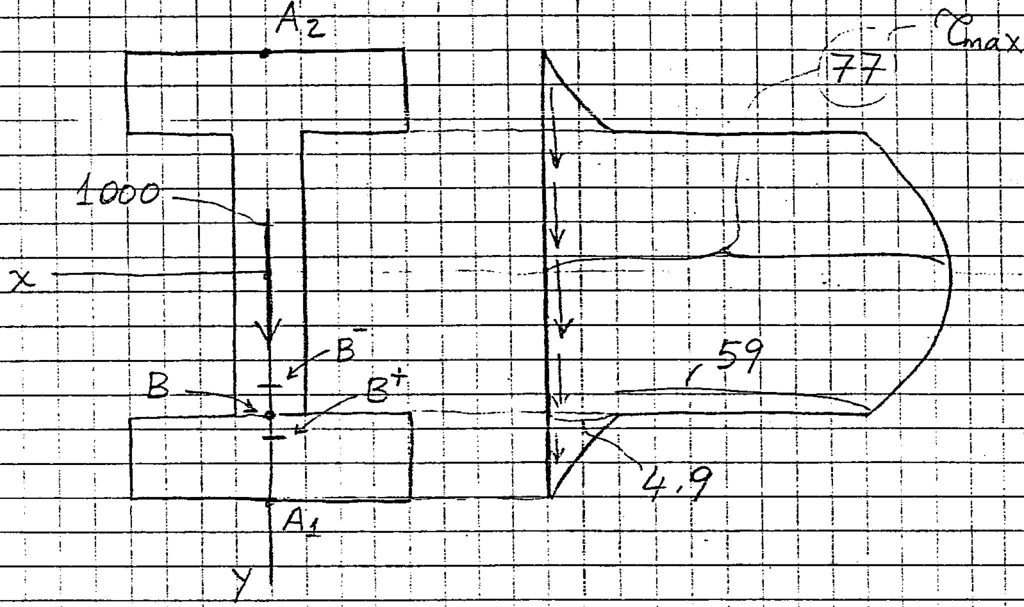
τ DAĞILIMI ?

$$\tau = \frac{\overset{1000}{T}\, S_x'}{b I_x}$$



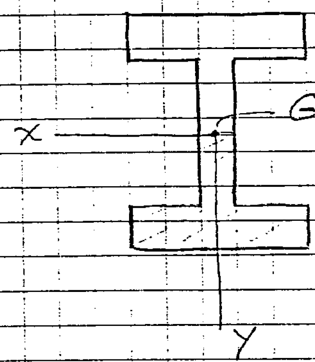
$$I_x = \frac{(0{,}75)(17{,}8)^3}{12} + 2 * \left[\frac{9 * (1{,}1)^3}{12} + (-9{,}45)^2 * 9 * 1{,}1\right]$$

$$\Rightarrow \boxed{I_x \cong 2122{,}7 \ cm^4}$$



$A_1, A_2 \rightarrow \tau = 0$

<u>G'de:</u>



$b = 0.75$

$$S_x' = (9*1.1)(9.45) + (0.75*8.9)*\frac{8.9}{2}$$

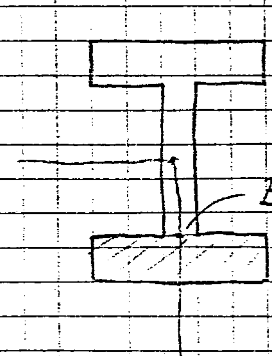
$$= 123.3\ cm^3$$

$$\Rightarrow \tau\Big|_G = \frac{(1000)(123.3)}{(0.75)(2122.7)} \cong \overset{\tau|_G}{\underset{\tau_{max}}{77\ kgf/cm^2}}$$

$B^+ \rightarrow$ B'nin hemen aşağısında

$B^- \rightarrow$ B'nin hemen yukarısında

$B^+, B^- \rightarrow S_x' = ?$



$$\Rightarrow \quad S_x' = (9)(1.1)(9.45)$$

$$= 93.6 \ cm^3$$

$B^+ \Rightarrow b = 9 cm$

$$\Rightarrow \tau\Big|_{B^+} = \frac{(1000)(93.6)}{(9)(2122.7)}$$

$$\Rightarrow \boxed{\tau\Big|_{B^+} \cong 4.9 \ kgf/cm^2}$$

$B^- \Rightarrow b = 0.75 \ cm$

$$\Rightarrow \tau\Big|_{B^-} = \frac{(1000)(93.6)}{(0.75)(2122.7)}$$

$$\Rightarrow \boxed{\tau\Big|_{B^-} \cong 59 \ kgf/cm^2}$$

$$\tau_{max} = k \frac{T}{F}$$ τ_{ort}

? (I profili için)

$$F = (9)(1.1)*2 + 17.8 * 0.75$$

$$= 33.15 \ cm^2$$

$$\Rightarrow \tau_{ort} = \frac{T}{F} = \frac{1000}{33.15} \cong 30.2 \ kgf/cm^2$$

$$\Rightarrow k = \frac{77}{30.2} \cong 2.55$$

I profilinde τ_{max}'ın hesabı için yaklaşık formül:

Yaklaşık formül elde etmek için:

1) Başlık kısımlarındaki kayma gerilmesini ihmal et
2) Gövde kısmındaki kayma gerilmesinin uniform olduğunu kabul et

$$\Rightarrow \boxed{\tau_{max} \cong \frac{T}{F_{gövde}}} \quad (*)$$

Bu örnek için (*) denklemini kullanarak τ_{max}'ın yaklaşık değerini bulalım:

$$F_{gövde} = (17.8)(0.75) = 13.35 \text{ cm}^2$$

$$\Rightarrow \tau_{max} \cong \frac{T}{F_{gövde}} = \frac{1000}{13.35}$$

$$\Rightarrow \boxed{\tau_{max} \cong 75 \text{ kgf/cm}^2}$$