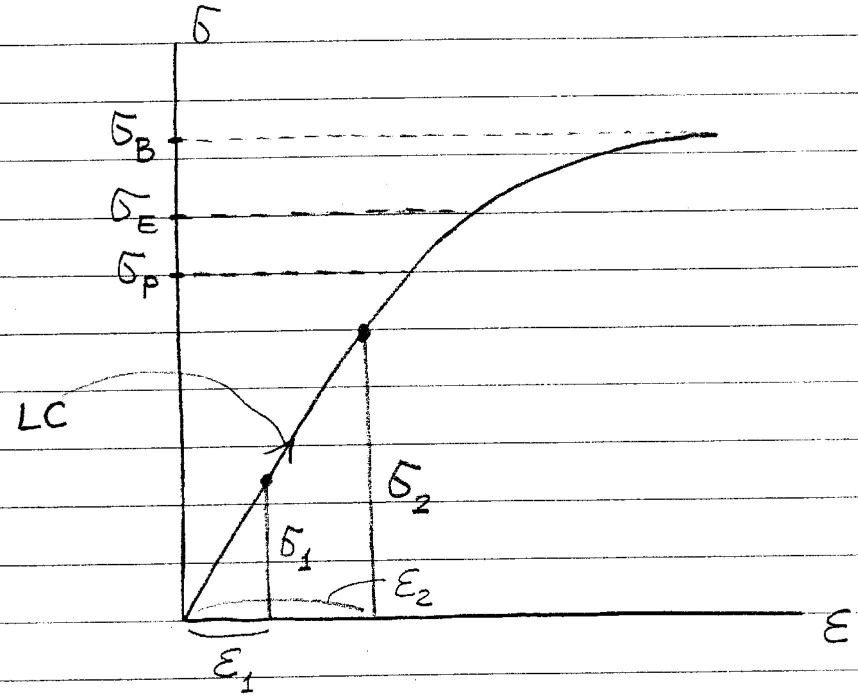
Now, mark the results in $(\varepsilon - \sigma)$ plane:



$0 \leq \sigma \leq \sigma_P \rightarrow$ the curve is linear

up to $\sigma_E \rightarrow$ the material remains elastic

$> \sigma_P$

$\Rightarrow$ LC & UC COINCIDE

$\sigma > \sigma_E \rightarrow$ the matl. becomes plastic

$\Rightarrow$ LC & UC DO NOT COINCIDE

at $\sigma = \sigma_B \rightarrow$ the material breaks up

$\sigma_P \rightarrow$ proportionality limit

$\sigma_E \rightarrow$ elastic limit

$> \sigma_P$

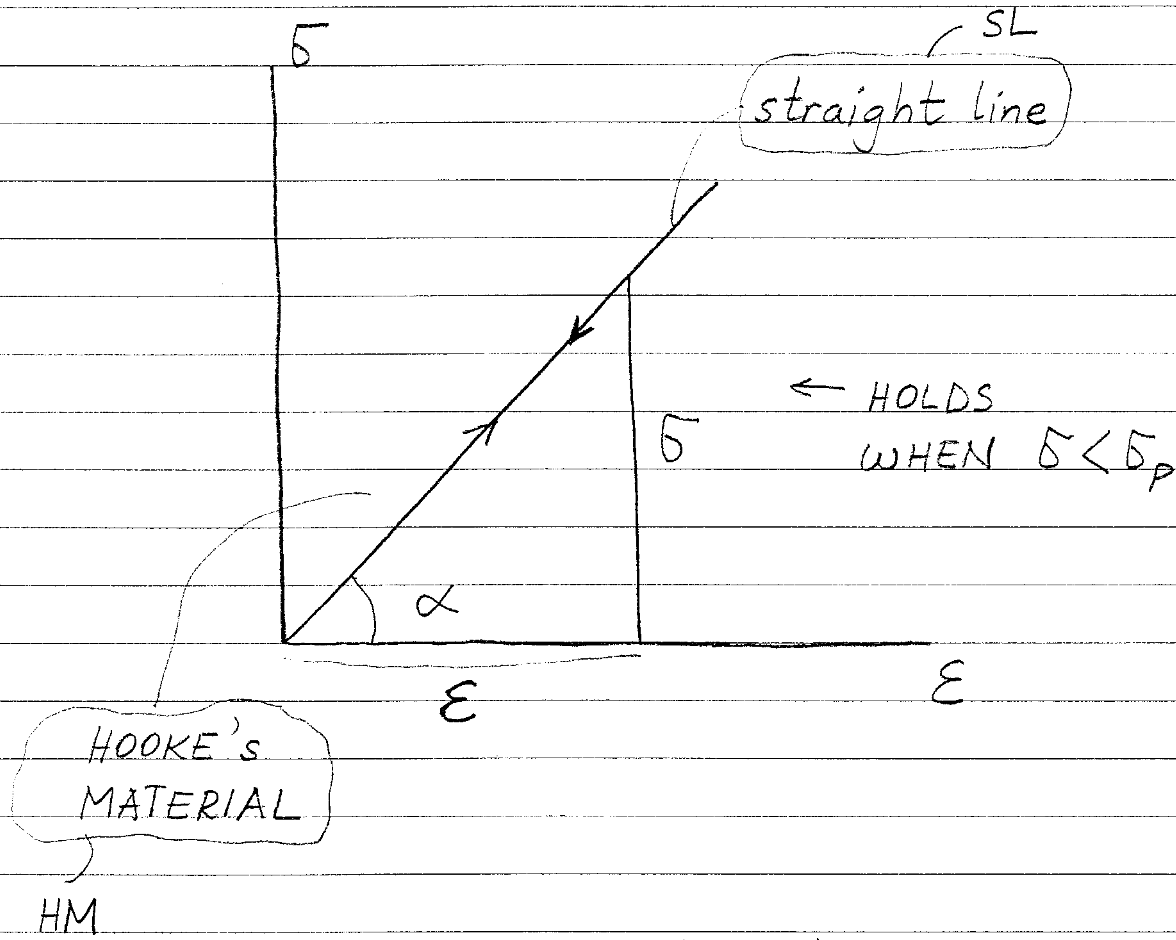
BS

$\sigma_B \rightarrow$ breaking stress (or strength)

In this course we will assume that:

$$\boxed{\sigma < \sigma_p}$$

$\Rightarrow$ the matl. will be both linear and elastic



Slope of SL → Young's or elasticity modulus & designated by **E**

$$\Rightarrow \quad \boxed{\tan\alpha = E}$$ elasticity modulus

$$\Rightarrow \quad \sigma = \underbrace{(\tan\alpha)}_{E} * \varepsilon$$

elasticity modulus

$$\Rightarrow \boxed{\sigma = E\,\varepsilon} \quad (*) \rightarrow$$ states that σ is proportional to ε

$\underbrace{\text{HOOKE's EQN.}}_{\text{HE}}$

$[\varepsilon] = ?$

$$\varepsilon = \frac{\Delta L}{L} \Rightarrow [\varepsilon] = \frac{[\Delta L]}{[L]} = \frac{L}{L}$$

(cm/cm, m/m, etc.)

$\Rightarrow [\varepsilon] \rightarrow$ nondimensional

$[E] = ?$

$$(*) \Rightarrow \sigma = E\varepsilon \Rightarrow \underbrace{[\sigma]}_{F/L^2} = [E]\underbrace{[\varepsilon]}_{L/L}$$

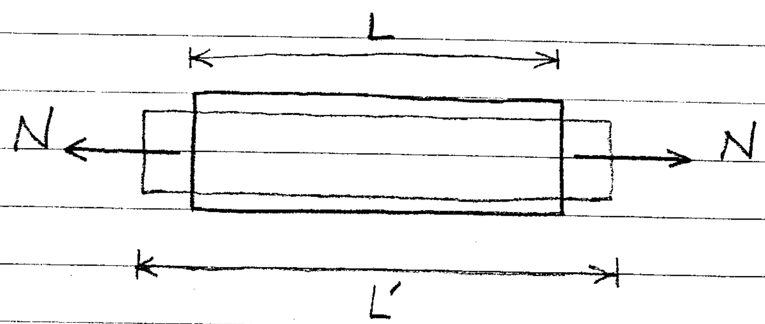
$$\Rightarrow \boxed{[E] = \frac{F}{L^2}}$$

kgf/cm², ton/m², etc.

The values of E for various materials

Material	E (kgf/cm^2)
Steel	$\sim 2-2.2 * 10^6$
Copper	$\sim 1.2 * 10^6$
Concrete	$\sim 2-3 * 10^5$
Aluminum	$\sim 0.7 * 10^6$
Wood	$\sim 1 * 10^5$

DETERMINATION OF ΔL IN TERMS OF AXIAL LOAD



$$\Delta L = L' - L = ? \text{ (in terms of } N\text{)}$$

$$\Delta L = \varepsilon * L \quad (1)$$

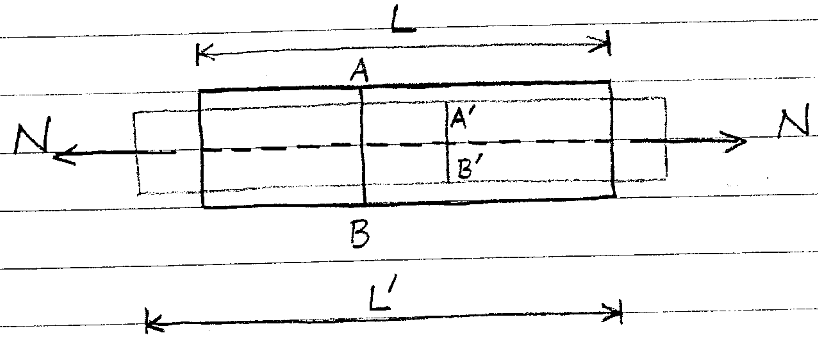
$$HE \Rightarrow \sigma = E\varepsilon \Rightarrow \varepsilon = \frac{\sigma}{E} \qquad \sigma = \frac{N}{A}$$

$$\Rightarrow \quad \boxed{\varepsilon = \frac{N}{EA}} \quad (2)$$

(2) → (1) :

$$\Rightarrow \quad \boxed{\Delta L = \frac{NL}{EA}}$$ determines ΔL in terms of N

LATERAL STRAIN



We have length changes in both axial and lateral directions.

The length change in axial direction was defined by the lon. stn. ε :

$$\underbrace{\varepsilon}_{\text{lon. stn.}} = \frac{\Delta L}{L}$$

Now, we will define the strain in lateral direction

$$\underbrace{\text{LAT. STN.}}_{\varepsilon_L}$$

$AB \perp$ Axis

$\Downarrow$

$A'B' \rightarrow AD$

$$\underbrace{\varepsilon_L}_{\text{LAT. STN.}} \triangleq \frac{\overline{A'B'} - \overline{AB}}{\overline{AB}}$$

If $N \rightarrow \oplus$ (T) $\Rightarrow \varepsilon > 0,\ \varepsilon_L < 0$

If $N \rightarrow \ominus \Rightarrow \varepsilon < 0,\ \varepsilon_L > 0$

Lon. Stn. ε

Lat. Stn. ε_L

have opposite signs

$$\Rightarrow \varepsilon * \varepsilon_L < 0$$

The experiments indicate that:

$$(*)\quad -\frac{\varepsilon_L}{\varepsilon} = \text{constant}$$

$\oplus$

The ratio (*) is called Poisson's ratio and designated by ν

$$\boxed{-\frac{\varepsilon_L}{\varepsilon} = \nu}$$

$[\nu] = ? \Rightarrow [\nu] = \dfrac{[\varepsilon_L]}{[\varepsilon]}$ nondim.

$\Rightarrow \nu \rightarrow$ nondim.

Theory of elasticity shows that:

$$0 \leq \nu \leq \underbrace{0.5}_{\substack{\text{corresponds to} \\ \text{incompressible matl.}}}$$

The values of ν for some materials:

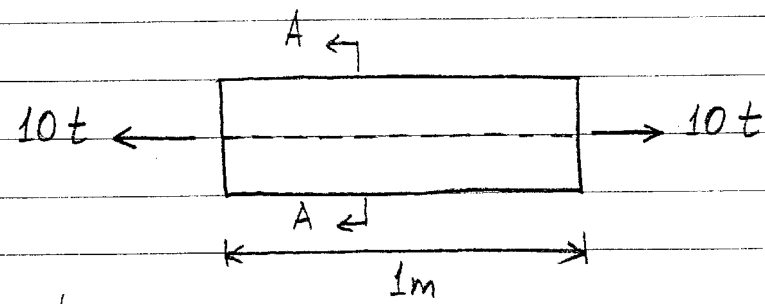
Material	ν
Steel	$\sim 0.25 - 0.30$
Copper	$\sim 0.31 - 0.34$
Concrete	~ 0.15

$$\left.\begin{array}{l} E \\ \nu \end{array}\right\} \underbrace{\text{material constants}}_{\text{The values of these constants change from one material to another}}$$

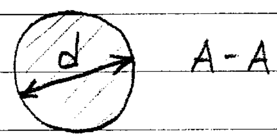
$$[E] \rightarrow F/L^2$$

$$[\nu] \rightarrow \text{nondim.}$$

EXAMPLE:



$F \to kgf$
$L \to cm$



a) determine "d" so that:

$$\Delta L \leq 0.01 \ cm \quad (*)$$

$$\underbrace{\sigma \leq \sigma_a}_{DE} \quad (**)$$

if $E = 2 * 10^6 \ kgf/cm^2$
$\sigma_a = 1000 \ kgf/cm^2$

b) For the value of "d" found in (a), compute ΔL and the amount of change in "d". Take $\nu = 0.3$

a) (*) :

$$\Delta L = \frac{\overset{10t}{N}\ \overset{1m}{L}}{EA} \leq 0.01 \ cm$$

$$\Rightarrow \frac{(10000)(100)}{2*10^6 * \underset{?}{A}} \leq 0.01$$

$$\Rightarrow \boxed{A \geqslant \frac{10^6}{2*10^4} = 50\ cm^2}$$

**:

$$\sigma = \frac{N}{A} = \frac{10000}{A} \leq \overset{1000}{\sigma_a}$$

$$\Rightarrow \boxed{A \geqslant 10\ cm^2}$$

both of them must be satisfied

$$\frac{\pi d^2}{4}$$

$$\Rightarrow A \geqslant 50\ cm^2$$

$$\Rightarrow d \geqslant \sqrt{\frac{200}{\pi}} = 7.98\ cm$$

$$\Rightarrow \text{CHOOSE}: \boxed{d = 10\ cm}$$

b) $\Delta L = ?\ ,\ \Delta d = ?$

$$\Delta L = \frac{NL}{EA}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi\,(10)^2}{4} \cong 78.5\ cm^2$$

$$\Rightarrow \Delta L = \frac{(10000)(100)}{(2*10^6)(78.5)} = \underbrace{0.006\ cm}$$

very small compared to $\underbrace{L}_{100\,cm}$

$$\Rightarrow \varepsilon = \frac{\Delta L}{L} = \frac{0.006\ cm}{100\ cm} \Rightarrow \boxed{\varepsilon = 0.6*10^{-4}\ \frac{cm}{cm}}$$

Lat. Stn.

$\varepsilon_L = ?$

$$-\frac{\varepsilon_L}{\varepsilon} = \nu \Rightarrow \varepsilon_L = -\nu\varepsilon$$

$$\Rightarrow \varepsilon_L = -(0.3)(0.6 * 10^{-4})$$

very small

$$\Rightarrow \boxed{\varepsilon_L = -0.18 * 10^{-4} \ cm/cm}$$

length decrease in lat. dir.

length change in "d"

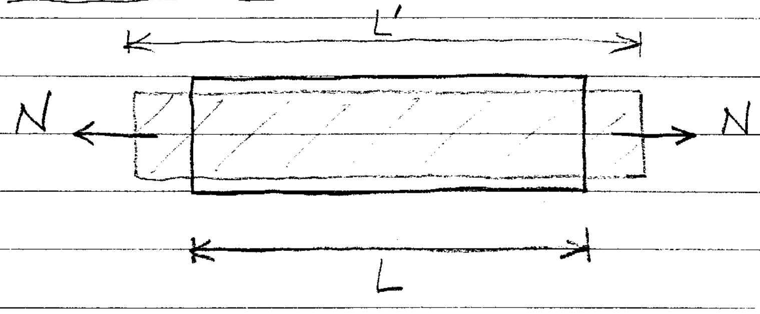
$$\Delta d = \varepsilon_L * d$$

$$= (-0.18 * 10^{-4})(\underbrace{10 \ cm}_{d})$$

$$\Rightarrow \boxed{\Delta d = -0.18 * 10^{-3} \ cm = -0.00018 \ cm}$$

length decrease in lat. dir.

very small



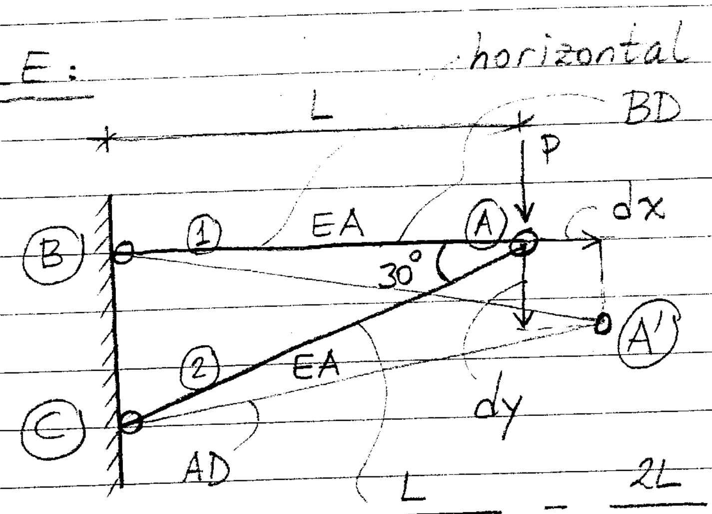
$$\Delta L = L' - L = \frac{NL}{EA} \Rightarrow \Delta L = \frac{NL}{EA} \quad (*)$$

EA is called AXIAL RIGIDITY (AR)

(*) $\Rightarrow$ As AR increases ΔL decreases

" " $\rightarrow \infty \Rightarrow \Delta L \rightarrow 0$

EXAMPLE:



$$\frac{L}{\cos 30^\circ} = \frac{2L}{\sqrt{3}}$$

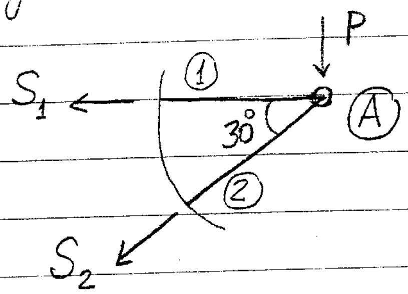
AR's of 1 & 2 are identical (EA)

wish to compute:

$$\left(\begin{array}{l} dx \rightarrow \text{hor. disp.} \\ dy \rightarrow \text{ver. } \text{"} \end{array} \right) \begin{array}{l} \text{of } A \text{ in terms} \\ \text{of } P, EA, L \end{array}$$

Compute bar forces:

Use joint method:



$$\overset{\downarrow}{+} \sum F_y = 0 \Rightarrow P + S_2 \sin 30^\circ = 0$$

$$\Rightarrow \boxed{S_2 = -2P \quad C}$$

$$+\leftarrow \sum F_x = 0 \Rightarrow S_1 + \underbrace{S_2}_{-2P} \cos 30^\circ = 0$$

$$\Rightarrow \boxed{S_1 = \sqrt{3}\,P \quad T}$$

Compute ΔL's in the bars 1 & 2:

Use $\Delta L = \dfrac{NL}{EA}$

$$1 \Rightarrow \left.\begin{array}{l} N = \sqrt{3}\,P \\ L = L \end{array}\right\} \Rightarrow \boxed{\Delta L_1 = \frac{(\sqrt{3}P)L}{EA}}$$

$+$ elongation E

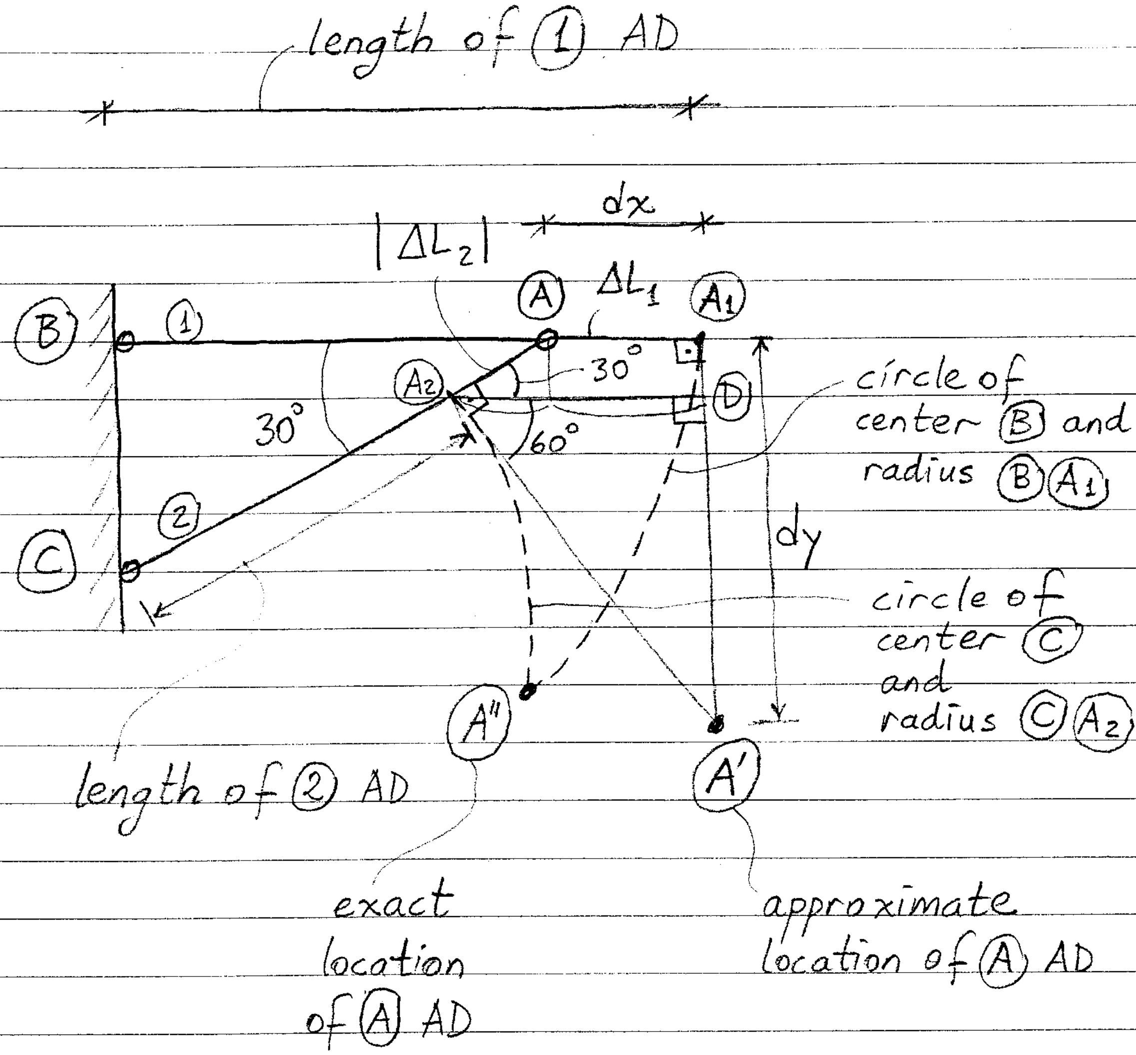
$$2 \Rightarrow \left.\begin{array}{l} N = -2P \\ L = 2L/\sqrt{3} \end{array}\right\} \Rightarrow \Delta L_2 = \frac{(-2P)\left(\frac{2L}{\sqrt{3}}\right)}{EA}$$

$$\Rightarrow \boxed{\Delta L_2 = \frac{-4PL}{\sqrt{3}\,EA}}$$

$-$ shortening S

Assume that ΔL_1, $|\Delta L_2|$ are small compared to L. To find the location of A' we will use geometrical arguments.

$\longrightarrow$



Since ΔL_1 and ΔL_2 are small, we approximate the circular arcs by tangent lines

$$\boxed{dx = \Delta L_1 = \frac{\sqrt{3}\,PL}{EA} \quad \rightarrow}$$

$$dy = \overline{A_1 D} + \overline{DA'}$$

$$\overline{A_1 D} = |\Delta L_2| \sin 30^\circ = \frac{|\Delta L_2|}{2}$$

$$\overline{DA'} = \overline{A_2 D}\ \underbrace{\tan 60^\circ}_{\sqrt{3}}$$

$$\overline{A_2D} = |\Delta L_2| \cos 30^\circ + \Delta L_1$$

$$= \frac{\sqrt{3}}{2} |\Delta L_2| + \Delta L_1$$

$$\Rightarrow \overline{DA'} = \frac{3}{2} |\Delta L_2| + \sqrt{3}\, \Delta L_1$$

$$\Rightarrow dy = \frac{|\Delta L_2|}{2} + \frac{3}{2} |\Delta L_2| + \sqrt{3}\, \Delta L_1$$

$$= 2 |\Delta L_2| + \sqrt{3}\, \Delta L_1$$

$$= \frac{8 PL}{\sqrt{3} EA} + \frac{3 PL}{EA}$$

$$\Rightarrow \boxed{dy = \left(\frac{8}{\sqrt{3}} + 3 \right) \frac{PL}{EA} \quad \downarrow}$$

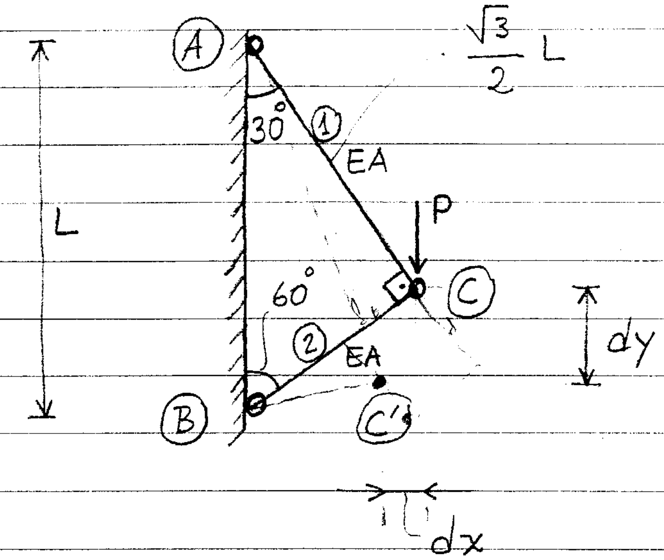
Numerical example:

$L = 1\,m$
$P = 1\,ton$
$A = 100\,cm^2$
$E = 10^5\,kgf/cm^2$

$$\Rightarrow \boxed{\begin{array}{l} dx = 0.017\,cm \\ \hline dy = 0.076\,cm \end{array}}$$

VERY SMALL
COMPARED TO
L = 100 cm

EXAMPLE :

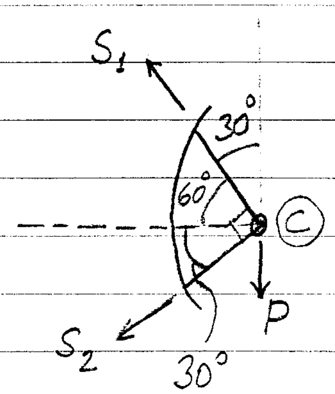


We wish to compute the displ. of C (in terms of P, L, EA)

dx, dy ?

Assume that the deformation is small.

Bar forces:



$$\overset{+}{\nwarrow} \sum F_{S_1} = 0 \Rightarrow S_1 - P * \underbrace{\cos 30^\circ}_{\sqrt{3}/2} = 0$$

$$\Rightarrow \boxed{S_1 = \frac{\sqrt{3}}{2} P \quad \text{(T)}}$$

$$\swarrow^{+} \sum F_{s_2} = 0 \Rightarrow S_2 + P * \overset{1/2}{\cos 60^\circ} = 0$$

$$\Rightarrow \boxed{S_2 = -\frac{P}{2} \quad C}$$

length changes in 1, 2:

$$\Delta L = \frac{NL}{EA}$$

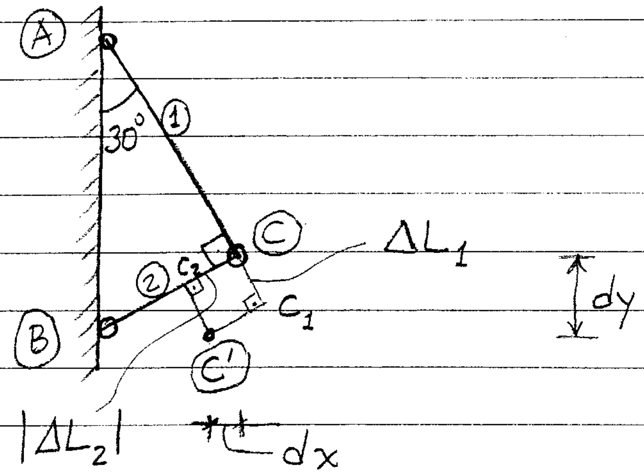
$$1 \Rightarrow \left.\begin{aligned} N &= +\frac{\sqrt{3}}{2}P \\ L &= \frac{\sqrt{3}}{2}L \end{aligned}\right) \Rightarrow \Delta L_1 = \frac{\left(\frac{\sqrt{3}}{2}P\right)\left(\frac{\sqrt{3}}{2}L\right)}{EA}$$

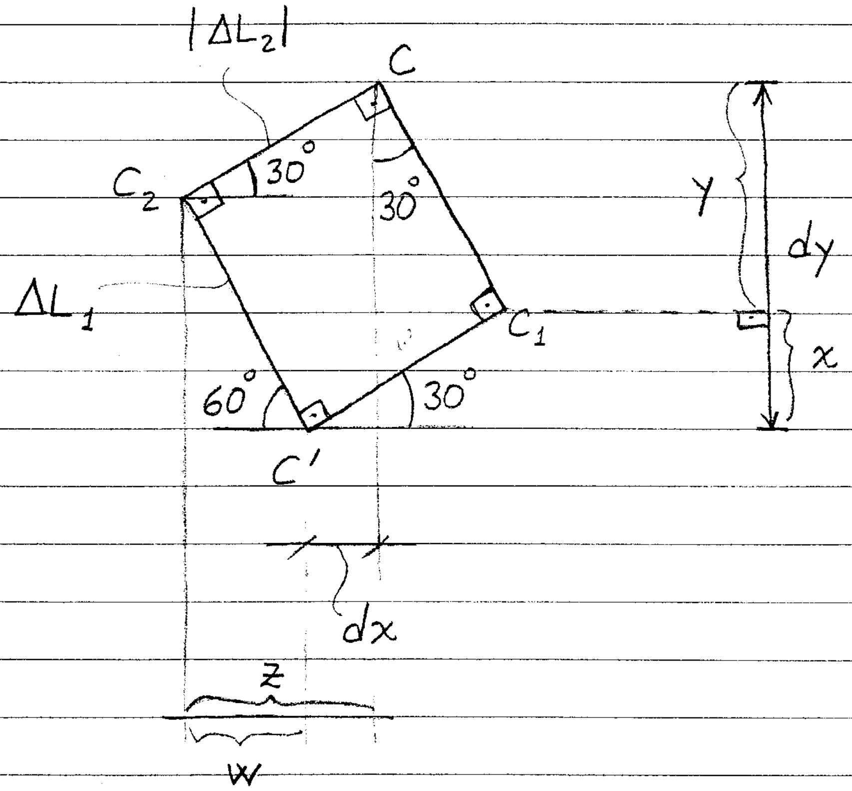
$$\Rightarrow \boxed{\Delta L_1 = \frac{3PL}{4EA} \quad E}$$

$$2 \Rightarrow \left.\begin{aligned} N &= -\frac{P}{2} \\ L &= L/2 \end{aligned}\right) \Rightarrow \Delta L_2 = \frac{\left(-\frac{P}{2}\right)\left(\frac{L}{2}\right)}{EA}$$

$$\Rightarrow \boxed{\Delta L_2 = -\frac{PL}{4EA} \quad S}$$

Geometrical considerations:





$$dy = \underbrace{x}_{|\Delta L_2|\sin 30^\circ} + \underbrace{y}_{\Delta L_1 \cos 30^\circ}$$

$$\underbrace{|\Delta L_2| \underbrace{\sin 30^\circ}_{1/2}}_{|\Delta L_2|/2} \qquad \underbrace{\Delta L_1 \overbrace{\cos 30^\circ}^{\sqrt{3}/2}}_{\frac{\sqrt{3}\,\Delta L_1}{2}}$$

$$\Rightarrow \quad dy = \frac{|\Delta L_2|}{2} + \frac{\sqrt{3}\,\Delta L_1}{2}$$

$$= \frac{PL}{8EA} + \frac{3\sqrt{3}\,PL}{8EA}$$

$$\Rightarrow \quad \boxed{dy = \frac{1+3\sqrt{3}}{8}\,\frac{PL}{EA} \;\downarrow}$$

$$dx = \underbrace{z}_{\underbrace{|\Delta L_2|\cos 30^\circ}_{\frac{\sqrt{3}\,|\Delta L_2|}{2}}} - \underbrace{W}_{\underbrace{\Delta L_1 \cos 60^\circ}_{\Delta L_1/2}}$$

$$\Rightarrow \quad dx = \frac{\sqrt{3}\,|\Delta L_2|}{2} - \frac{\Delta L_1}{2}$$

$$= \frac{\sqrt{3}\,PL}{8EA} - \frac{3PL}{8EA}$$

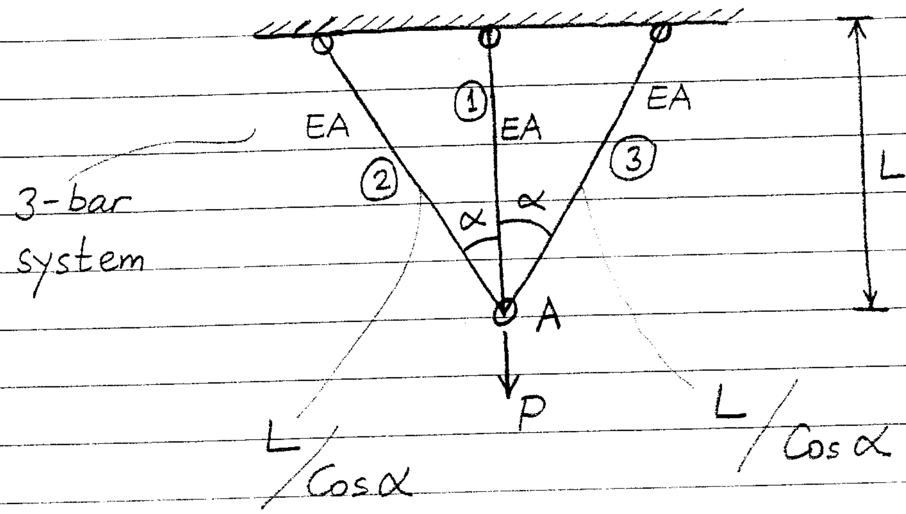
$$\ominus$$

$$= \frac{(\sqrt{3}-3)}{8}\,\frac{PL}{EA}$$

$$\Rightarrow \quad \boxed{dx = \frac{(3-\sqrt{3})}{8}\,\frac{PL}{EA} \;\rightarrow}$$

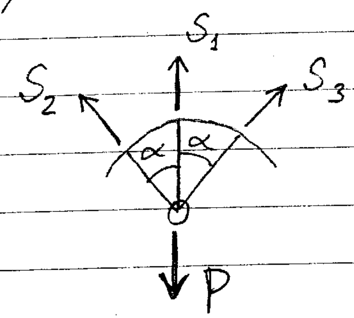
INDETERMINATE SYSTEMS

EXAMPLE:



we wish to find bar forces in terms of P.

First, attempt to compute bar forces by EE:



$$\xrightarrow{+} \sum F_x = 0 \Rightarrow S_3 \sin\alpha - S_2 \sin\alpha = 0$$

$$\Rightarrow \boxed{S_3 = S_2} \quad (1)$$

$$\overset{+}{\uparrow} \sum F_y = 0 \Rightarrow S_1 + S_2 \cos\alpha + \underbrace{S_3}_{S_2} \cos\alpha - P = 0$$

$$\Rightarrow \boxed{S_1 + 2S_2 \cos\alpha = P} \quad (2)$$

moment EE will be identically satisfied since all of forces pass through the pt. A.

$\underbrace{\text{available eqs.}}_{}$ AE

$\underbrace{\text{Unknowns}}_{}$ U

(1) $\Rightarrow$ 1

(2) $\Rightarrow$ $\underline{1}$

2

$\left.\begin{matrix} S_1 \\ S_2 \\ S_3 \end{matrix}\right\} \Rightarrow 3$

$\Rightarrow$ to find bar f. we need 1 more eqn.

$$\underbrace{3}_{\underbrace{\#\text{ of U}}_{n_1}} - \underbrace{2}_{\underbrace{\#\text{ of AE}}_{n_2}} = \underbrace{1}_{\underbrace{\#\text{ of eqs. we need}}_{n}}$$

$\underbrace{n}_{n_1 - n_2} \rightarrow \underbrace{\text{degree of indeterminacy}}_{DI}$

Note that $n = 0$ for determinate sys.
In our problem $DI = 1$

The additional eqn. can be obtained by taking into account the deformation of the system, the additional eqn. last obtained is called "compatibility eqn." (CE).

Now, let us find CE for our problem:

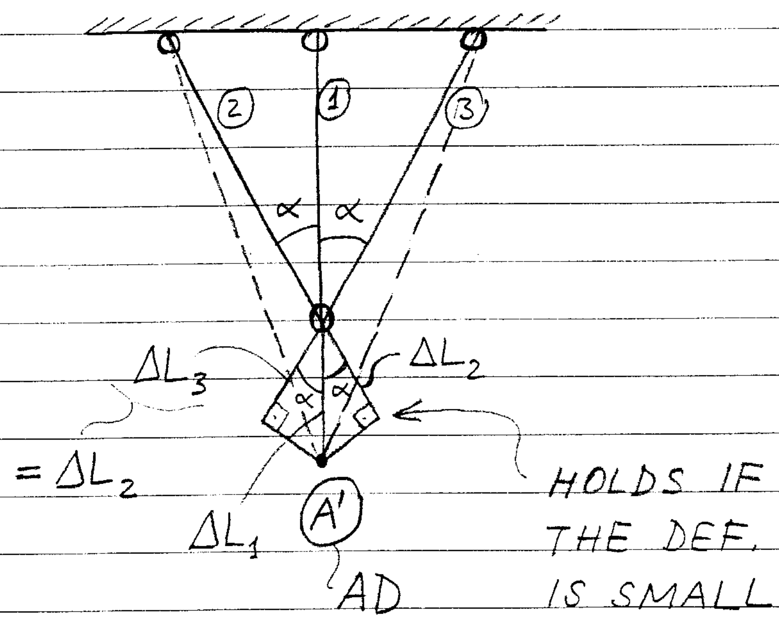
length changes in the bars:

$$\Delta L = \frac{NL}{EA}$$

$$① \Rightarrow \left.\begin{matrix} L \to L \\ N \to S_1 \end{matrix}\right\} \Rightarrow \Delta L_1 = \frac{S_1 L}{EA}$$

$$② \Rightarrow \left.\begin{matrix} L \to L/\cos\alpha \\ N \to S_2 \end{matrix}\right\} \Rightarrow \Delta L_2 = \frac{S_2 * \frac{L}{\cos\alpha}}{EA}$$

$$\Rightarrow \Delta L_2 = \frac{S_2 L}{EA\cos\alpha} = \Delta L_3$$



From the fig:

$$\Rightarrow \boxed{\underbrace{\Delta L_2}_{\frac{S_2 L}{EA\cos\alpha}} = \underbrace{\Delta L_1}_{\frac{S_1 L}{EA}} * \cos\alpha}$$

CE in terms of length changes

$$\Rightarrow \boxed{S_2 = S_1 \cos^2\alpha} \quad (3)$$

CE in terms of bar f.

This is the eqn. which we need to solve the problem

(3) → (2):

$$\Rightarrow S_1 + 2 * \overbrace{\left(S_1 \cos^2\alpha\right)}^{S_2} * \cos\alpha = P$$

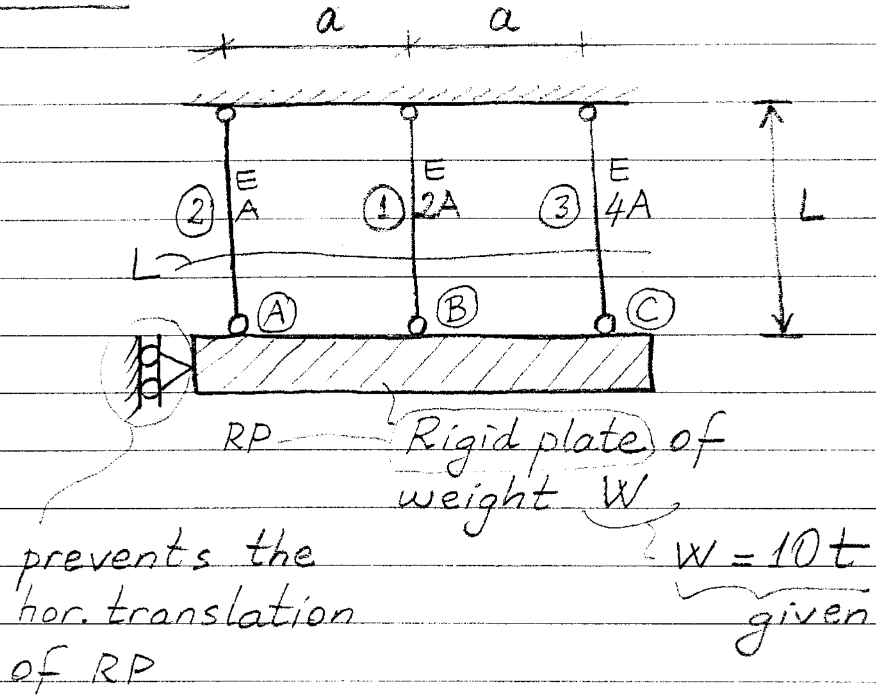
$$\Rightarrow \boxed{S_1 = \frac{P}{1 + 2\cos^3\alpha}} \quad (*)$$

bar f.

(*) → (3):

$$\Rightarrow \boxed{S_2 = \frac{P\cos^2\alpha}{1 + 2\cos^3\alpha} = S_3}$$

EXAMPLE :

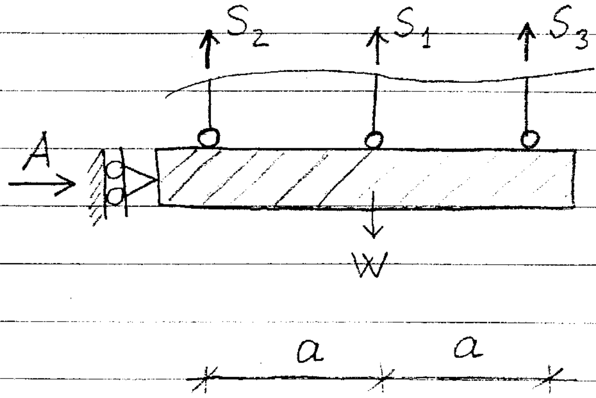


a) we wish to find bar f. in terms of W

b) Given $\sigma_a = 1200\ kgf/cm^2$ we wish to determine CSA "A"

c) Given $E = 2*10^6\ kgf/cm^2$, $L = 1\,m$ we wish to find the ver. displ. of "B".

a)



EE :

$$\xrightarrow{+} \sum F_x = 0 \quad \Rightarrow \quad \boxed{A = 0}$$

$$\overset{+}{\uparrow} \sum F_y = 0 \Rightarrow S_1 + S_2 + S_3 - W = 0$$

$$\Rightarrow \boxed{S_1 + S_2 + S_3 = W} \quad *$$

$$\overset{+}{\circlearrowleft} \sum M_B = 0 \Rightarrow S_3 * \cancel{a} - S_2 * \cancel{a} = 0$$

$$\Rightarrow \boxed{S_3 = S_2} \quad (1)$$

$$* \Rightarrow \boxed{S_1 + 2S_2 = W} \quad (2)$$

AE	U
(1) $\Rightarrow$ 1	S_1
(2) $\Rightarrow$ 1	S_2
2	S_3

$\Rightarrow 3$

$\Rightarrow DI = 3 - 2 = 1$

$\Rightarrow$ we need one CE to find bar f.

Now, find CE:

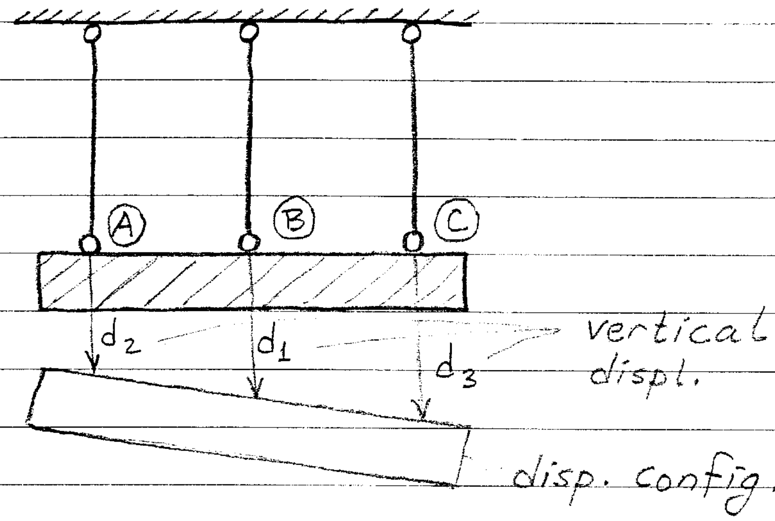
ΔL's in the bars: $\left(\Delta L = \dfrac{NL}{EA}\right)$

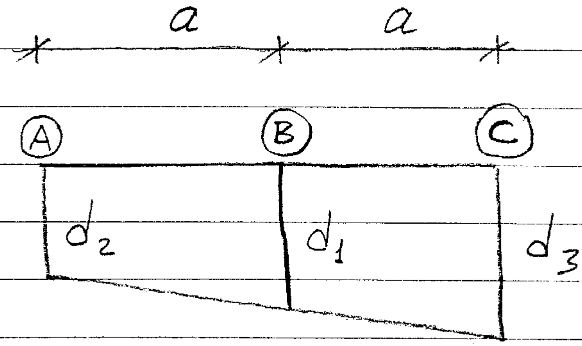
$$(1) \Rightarrow \left.\begin{array}{l} L \rightarrow L \\ E \rightarrow E \\ A \rightarrow 2A \\ N \rightarrow S_1 \end{array}\right\} \Rightarrow \boxed{\Delta L_1 = \dfrac{S_1 L}{E * 2A}}$$

(2) $\Rightarrow$ $$\boxed{\Delta L_2 = \frac{S_2 L}{EA}}$$

(3) $\Rightarrow$ $$\boxed{\Delta L_3 = \frac{S_3 L}{E * 4A} = \frac{S_2 L}{4EA}}$$

Displaced configuration:





$$\Rightarrow \underbrace{d_1}_{\Delta L_1} = \frac{1}{2}\left(\underbrace{d_2}_{\Delta L_2} + \underbrace{d_3}_{\Delta L_3}\right)$$

$$\Rightarrow \underbrace{\boxed{\Delta L_1 = \frac{1}{2}\left(\Delta L_2 + \Delta L_3\right)}}_{\text{CE in terms of } \Delta L\text{'s}}$$

$$\Rightarrow \frac{S_1 L}{2EA} = \frac{1}{2}\left(\frac{S_2 L}{EA} + \frac{S_2 L}{4EA}\right)$$

$$\Rightarrow \underbrace{\boxed{S_1 = \frac{5}{4} S_2}}_{\text{CE in terms of bar f.}} \quad (3)$$

(3) → (2) :

$$\Rightarrow \frac{5}{4} S_2 + 2S_2 = W$$

$$\Rightarrow \boxed{\underbrace{S_2}_{= S_3} = \frac{4}{13} W \cong 0.31 \; W} \quad (*)$$

(*) → (3) :

$$\Rightarrow S_1 = \frac{5}{4} * \frac{4}{13} W$$

$$\Rightarrow \boxed{S_1 \cong 0.38 \; W}$$

b) $\sigma_a = 1200 \; kgf/cm^2$

$W = 10 \text{ tons} = 10000 \; kgf$

$\boxed{\begin{array}{c} F \rightarrow kgf \\ L \rightarrow cm \end{array}}$

$A = ?$

DE for 1:

$$\sigma = \frac{S_1}{2A} \leq \sigma_a$$

$$\Rightarrow A \geqslant 1.58\ cm^2 \quad (*)$$

DE for 2:

$$\sigma = \frac{S_2}{A} \leq 1200$$

$$\Rightarrow A \geqslant 2.58\ cm^2 \quad (**)$$

There is no need for checking bar 3 since:

$$S_3 = S_2 \quad \text{and} \quad A_{3} = 4\ A_{2}$$

$\Rightarrow$ Both of (*) and (**) must be satisfied

$$\Rightarrow \boxed{\begin{array}{l} A \geqslant 2.58\ cm^2 \\ \text{CHOOSE} : A = 4\ cm^2 \end{array}}$$

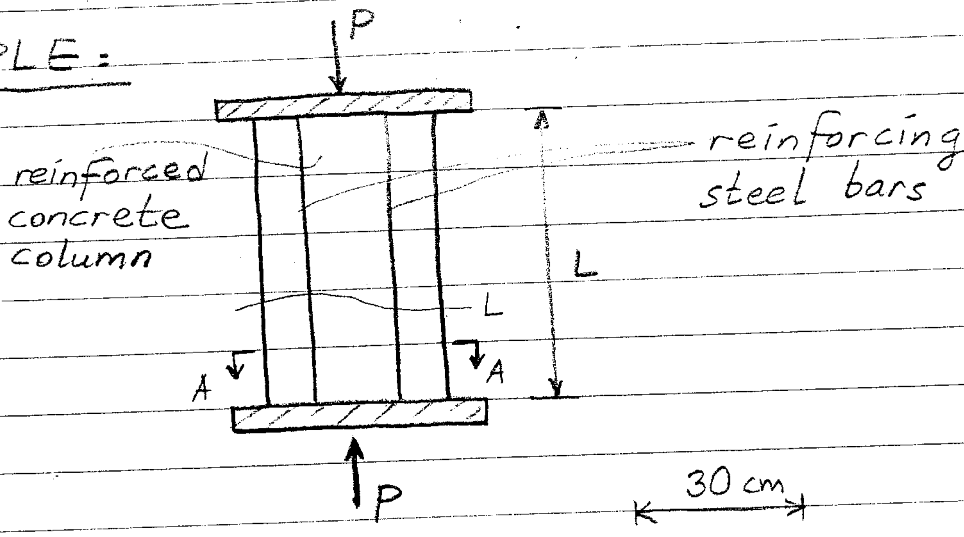
c) $E = 2 * 10^6\ kgf/cm^2$
$L = 1\ m = 100\ cm$
$d_1 = ?$

$$d_1 = \Delta L_1 = \frac{\overbrace{S_1}^{3800}\ \overbrace{L}^{100}}{\underbrace{E}_{2*10^6} * \underbrace{2A}_{8}}$$

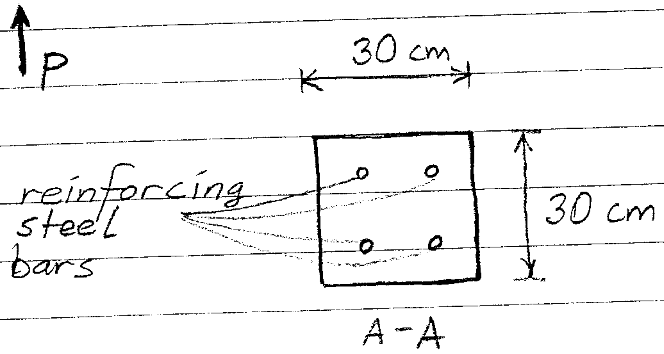
$$\Rightarrow \boxed{d_1 \cong 0.024\ cm}$$ ~ vertical displ. of "B"

very small compared to L = 10

EXAMPLE:



subscript "c" → concrete
subscript "s" → steel



$$A_s = 10 \ cm^2$$

$$\frac{E_s}{E_c} = 15$$

$$(\sigma_a')_c = 225 \ kgf/cm^2$$

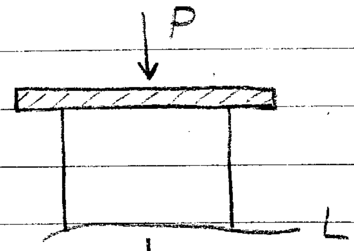
$$(\sigma_a')_s = 2100 \ kgf/cm^2$$

wish to find max value of P which the column can carry safely.

Area of concrete part:

$$A_c = 30 * 30 - 10$$

$$\Rightarrow \boxed{A_c = 890 \ cm^2}$$



$\downarrow N_s \longrightarrow$ axial f. carried by steel

$\downarrow N_c \longrightarrow$ axial f. carried by concrete

EE :

$$\overset{\downarrow}{+}\sum F_y = 0 \Rightarrow N_s + N_c + P = 0$$

$$\Rightarrow \boxed{N_s + N_c = -P} \quad (1)$$

AE	U
(1) $\Rightarrow$ 1	N_s
1	N_c

$\Rightarrow 2$

$$\Rightarrow DI = 2 - 1 = 1$$

$\Rightarrow$ we need one more eqn. which will come from CE.

CE:

$$\Delta L_s = \Delta L_c$$

"the amount of length changes in 's' and 'c' should be the same

$$\Delta L = \frac{NL}{EA} \Rightarrow \frac{N_s \cancel{L}}{E_s A_s} = \frac{N_c \cancel{L}}{E_c A_c}$$

$$\Rightarrow N_s = \frac{\overbrace{E_s}^{15}}{E_c} * \frac{A_s}{A_c} N_c$$

$$\Rightarrow \boxed{N_s = 0.169\ N_c} \quad 2$$

CE in terms of forces

2 → 1 :

$$\Rightarrow 0.169 N_c + N_c = -P$$

$$\Rightarrow \boxed{N_c = -0.855\ P \quad C}$$

$$\Rightarrow \boxed{N_s = -0.145\ P \quad C}$$

Note that:

$$N_s + N_c = -P$$

DE for concrete:

$$|\sigma| = \frac{|N_c|}{A_c} \leq (\sigma_a')_c$$

$$\Rightarrow P \leq 234210 \text{ kgf} \quad (*)$$

DE for steel:

$$|\sigma| = \frac{|N_s|}{A_s}$$

$$\Rightarrow P \leq 144827 \text{ kgf} \quad (**)$$

Both (*) & (**) must be satisfied

$$\Rightarrow \boxed{P_{max} = 144827 \text{ kgf} \cong 145 \text{ tons}}$$