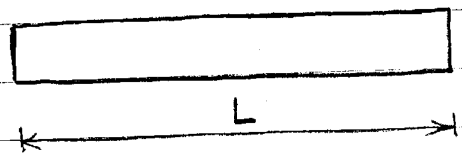
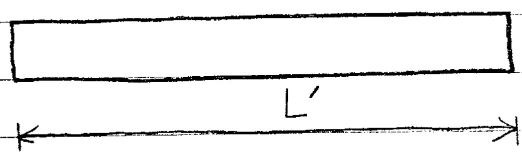
THE EFFECT OF TEMPERATURE CHANGES (THERMAL EFFECTS)



at $\underbrace{T}_{\text{temperature (TEMP)}} = \underbrace{T_0}_{\text{initial TEMP.}}$

suppose that the TEMP is changed by the amount ΔT:

$$\Rightarrow \underbrace{T}_{\text{the present TEMP.}} = \underbrace{T_0}_{\text{initial TEMP.}} + \Delta T$$



at $T = T_0 + \Delta T$

$\Delta L = L' - L \rightarrow$ length change produced by ΔT

Experiments show that:

$$\boxed{\Delta L = \alpha * \Delta T * L}$$

$\alpha \rightarrow$ $\underbrace{\text{thermal expansion coefficient}}_{\text{TEC}}$

length change in the unit length of the bar caused by unit temp. change

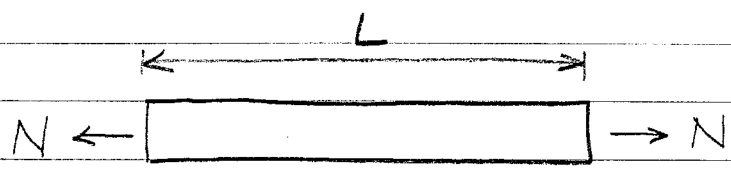
$[\alpha] = ?$

$$\underbrace{[\Delta L]}_{L} = [\alpha]\underbrace{[L]}_{L}\underbrace{[\Delta T]}_{\underbrace{^\circ C}_{\text{centigrade}}}$$

$$\Rightarrow \quad [\alpha] = \frac{L}{L} * \frac{1}{^\circ C}$$

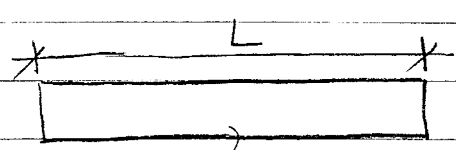
$$\rightarrow \quad \frac{cm}{cm} * \frac{1}{^\circ C}$$

$$\frac{m}{m} * \frac{1}{^\circ C}$$



$T = T_0 + \Delta T$

$$\Rightarrow \underbrace{\boxed{\Delta L = \frac{NL}{EA} + \alpha * \Delta T * L}}_{\text{length change when the bar is subjected to both } N \,\&\, \Delta T}$$

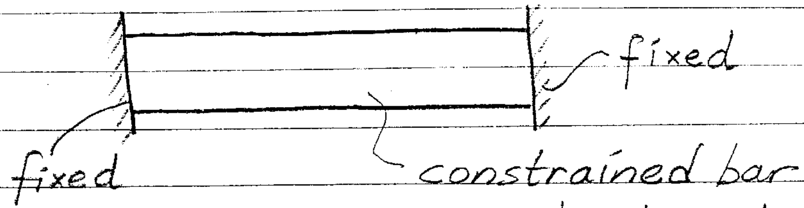


free to extend → free bar

$T = T_0 + \Delta T$ → produces no forces in the bar since the bar free to extend.

↓ PRODUCES

$\Delta L = \alpha * \Delta T * L$



↳ its extension is prevented

$T = T_0 + \Delta T$ $\quad \Rightarrow \Delta L = 0$

$$\Rightarrow \quad \underbrace{\Delta L}_{0} = \frac{NL}{EA} + \alpha * \Delta T * L$$

$$\Rightarrow \quad \boxed{N = -EA\,\alpha\,\Delta T}$$

compressional f. produced by ΔT.

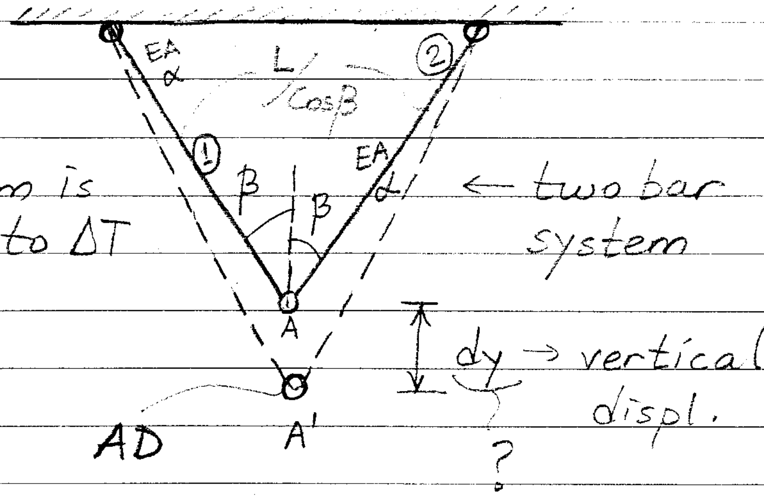
ΔT { produces no forces if the element is free to extend

produces forces if the element is constrained (not free to extend)

ΔT does not produce any internal forces in determinate systems — DS since the elements of DS are free to extend

---

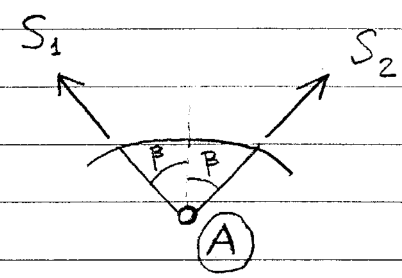
This system is subjected to ΔT



← two bar system

wish to compute bar f. and the displ. of A.

Isolate the pt. A :



EE :

$$\xrightarrow{+} \sum F_x = 0$$

$$\Rightarrow S_2 \sin\beta - S_1 \sin\beta = 0$$

$$\Rightarrow \boxed{S_2 = S_1} \quad (1)$$

$$+\uparrow \sum F_y = 0$$

$$\Rightarrow S_1 \cos\beta + S_2 \cos\beta = 0$$

$$\Rightarrow \boxed{S_1 + S_2 = 0} \quad (2)$$

AE	U
1 → 1	S_1
2 → 1	S_2
2	2

$\Rightarrow DI = 2 - 2 = 0$

$\Rightarrow$ The sys. is determinate

$\Rightarrow$ # of AE = # of U

$1 \to 2 \Rightarrow 2S_1 = 0 \Rightarrow \boxed{S_1 = 0}$

$1 \Rightarrow \boxed{S_2 = 0}$

since the sys. is determinate

$$\Delta L = \frac{NL}{EA} + \alpha \Delta T L$$

ΔL's for the bars:

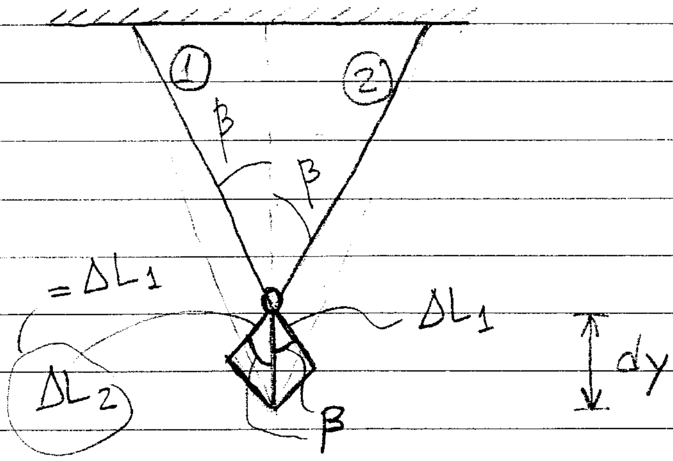
$$1 \Rightarrow \Delta L_1 = 0 + \alpha * \Delta T * \frac{L}{\cos\beta}$$

$$\Rightarrow \boxed{\Delta L_1 = \frac{\alpha * \Delta T * L}{\cos\beta}}$$

=

$$2 \Rightarrow \boxed{\Delta L_2 = \frac{\alpha * \Delta T * L}{\cos\beta}}$$

$*$



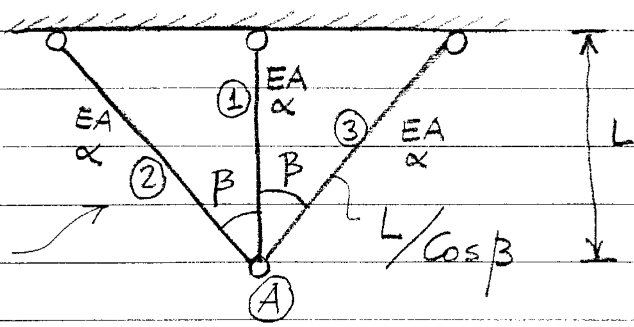
$$\Rightarrow \boxed{dy = \frac{\Delta L_1}{\cos\beta} = \frac{\alpha * \Delta T * L}{\cos^2\beta}}$$

The temp. change causes internal f. in indeterminate systems (IS) since the elements of IS are not free to extend.

EXAMPLE:

$\alpha \rightarrow$ TEC

3-bar sys

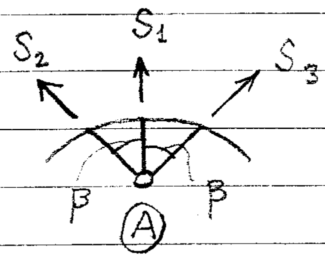


↓

Subjected to temp. change ΔT.

wish to find bar f.

isolate A:



EE:

$$\xrightarrow{+} \sum F_x = 0$$

$$\Rightarrow S_3 \sin\beta - S_2 \sin\beta = 0$$

$$\Rightarrow \boxed{S_3 = S_2} \quad (1)$$

$$+\uparrow \sum F_y = 0$$

$$\Rightarrow S_1 + S_2 \cos\beta + \overset{S_2}{S_3} \cos\beta = 0$$

$$\Rightarrow \boxed{S_1 + 2S_2 \cos\beta = 0} \quad (2)$$

AE	U
(1) → 1	S_1
(2) → 1	S_2
2	S_3

$S_1, S_2, S_3 \Rightarrow 3$

DI = 3 − 2 = 1 ⇒ the sys. is IS.

⇒ need one more eqn. which will come from CE.

Compute ΔL's:

$$\left(\Delta L = \frac{NL}{EA} + \alpha \Delta T L \right)$$

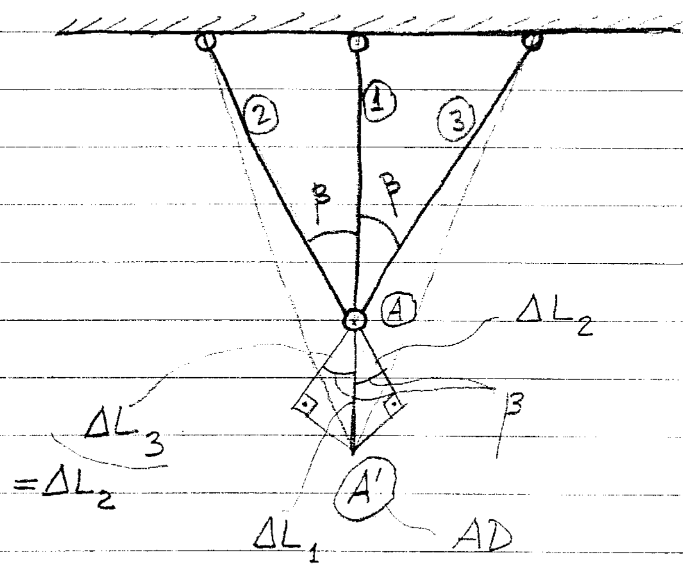
① $\Rightarrow \quad \Delta L_1 = \dfrac{S_1 L}{EA} + \alpha \Delta T L$

② $\Rightarrow \quad \Delta L_2 = \dfrac{S_2 (L/\cos\beta)}{EA} + \alpha \Delta T \dfrac{L}{\cos\beta}$

$$\Rightarrow \quad \Delta L_2 = \frac{S_2 L}{EA \cos\beta} + \frac{\alpha \Delta T L}{\cos\beta}$$

③ $\Rightarrow \quad \Delta L_3 = \Delta L_2$

CE:



From the fig:

$$\Delta L_2 = \Delta L_1 * \cos\beta$$ ← CE in terms of ΔL's

$$\Rightarrow \frac{S_2 L}{EA\cos\beta} + \frac{\alpha \Delta T L}{\cos\beta} = \left(\frac{S_1 L}{EA} + \alpha\Delta T L\right)\cos\beta$$

$$\Rightarrow S_2 + \alpha\Delta T EA = (S_1 + \alpha\Delta T EA)\cos^2\beta$$

$$\Rightarrow S_2 - S_1\cos^2\beta = \underbrace{(\cos^2\beta - 1)}_{-\sin^2\beta}\alpha\,\Delta T EA$$

$$\Rightarrow \boxed{S_1\cos^2\beta - S_2 = \sin^2\beta(\alpha\Delta T EA)} \quad (3)$$

CE in terms of bar f.

Solve (2) & (3) for S_1 & S_2 :

(2) $\Rightarrow$ $\boxed{S_1 = -2S_2\cos\beta}$ (*)

(*) $\rightarrow$ (3) :

$$\Rightarrow -2S_2\cos^3\beta - S_2 = EA\alpha\Delta T\sin^2\beta$$

$$\Rightarrow \boxed{S_2 = \frac{-\sin^2\beta}{2\cos^3\beta + 1} * EA\alpha\Delta T = S_3} \quad (C)$$

(*) :

$$\Rightarrow \boxed{S_1 = \frac{2\sin^2\beta\cos\beta}{2\cos^3\beta + 1} * EA\alpha\Delta T} \quad (T)$$

Interpretation:

$$\underbrace{L_2 = L_3}_{L/\cos\beta} > \overbrace{L_1}^{L}$$

If the bars were free to extend:

$(\Delta L = \alpha \Delta T L)$

$$\Downarrow$$

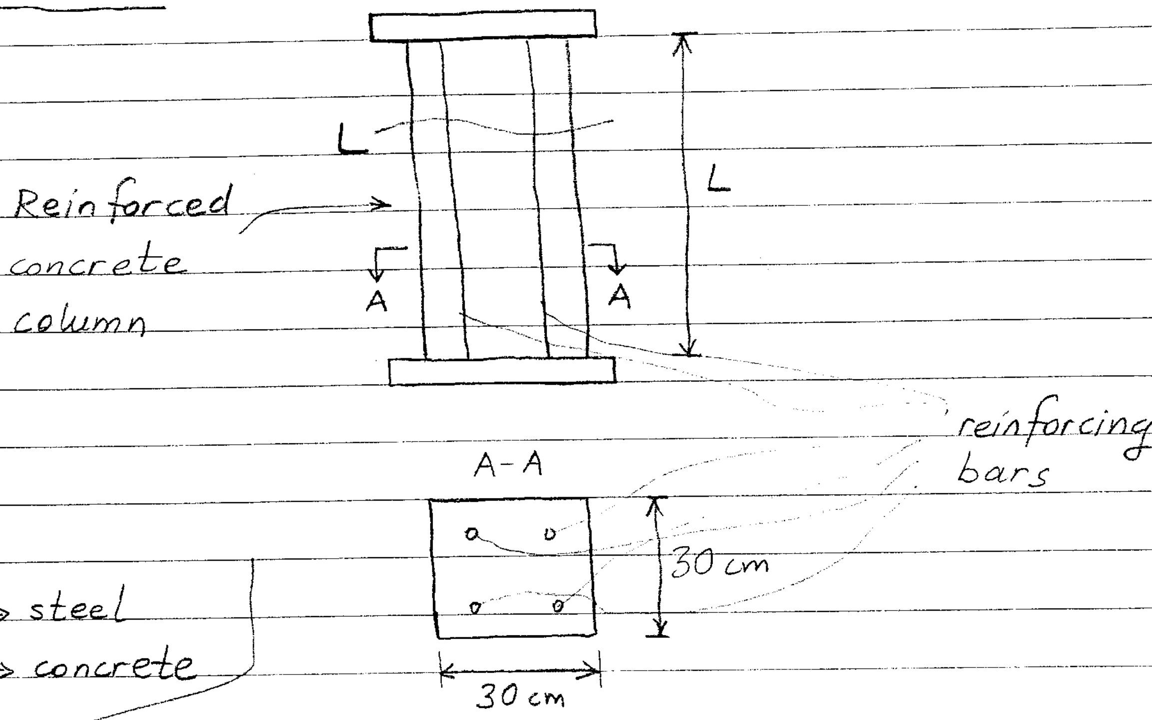
$$\Delta L_2 = \Delta L_3 > \Delta L_1$$

But, the bars are not free to extend, because they are connected to each other at A.

For this reason the bar 1 will try to prevent the free extensions of the bars 2 and 3. This will produce compressional forces in the bars 2 and 3.

The opposite situation holds for the bar 1, which resulting tensile forces in that bar.

EXAMPLE :



s → steel

c → concrete

$$\alpha_s = 12 * 10^{-6} \frac{cm}{cm} * \frac{1}{°C}$$

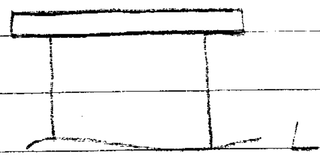
$$\alpha_c = 9 * 10^{6} \quad '' \quad ''$$

$$\frac{E_s}{E_c} = 15 \,, \quad A_s = 10\ cm^2 , \quad \Delta T = 50°C$$

$$E_s = 2 * 10^6\ kgf/cm^2$$

wish to compute the stresses in steel and concrete produced by the temp. increase $\Delta T = 50°C$

$$\Rightarrow \quad A_c = 890\ cm^2$$



$\downarrow N_c \leftarrow$ axial load in "c"

$\downarrow N_s \leftarrow$ " " " "s"

EE:

$$\overset{\downarrow}{+}\sum F_y = 0 \Rightarrow \boxed{N_c + N_s = 0} \quad (1)$$

$$\Rightarrow N_s = -N_c$$

AE	U
(1) → 1	N_c
	N_s
1	2

$$\Rightarrow \underbrace{DI = 2 - 1 = 1}_{\text{need one more eqn.}} \Rightarrow \text{The system is indeterminate}$$

need one more eqn.
↓
CE

The system is indeterminate
⇓
ΔT will produce forces in "s" and "c"

CE:

$$\Delta L_s = \Delta L_c \leftarrow \text{since "c" and "s" are bounded to each other}$$

CE in terms of ΔL's

$$\Rightarrow \frac{N_s * \cancel{L}}{E_s A_s} + \alpha_s \Delta T \cancel{L} = \frac{N_c \cancel{L}}{E_c A_c} + \alpha_c \Delta T \cancel{L}$$

$$\Rightarrow N_s + E_s A_s * \alpha_s \Delta T = \frac{E_s}{E_c} * \frac{A_s}{A_c} N_c + E_s A_s \alpha_c \Delta T$$

$$\Rightarrow \boxed{\underbrace{N_s}_{=-N_c \leftarrow (1)} - 0.169\, N_c = E_s A_s \Delta T (\alpha_c - \alpha_s)} \quad (2)$$

CE in terms of forces

Solve (1) and (2) for N_s and N_c:

$$\Rightarrow \boxed{\begin{aligned} N_c &\cong 2566 \text{ kgf} \quad (T) \\ N_s &= -N_c = -2566 \text{ kgf} \quad (C) \end{aligned}}$$

Now compute the stresses in "c" and "s":

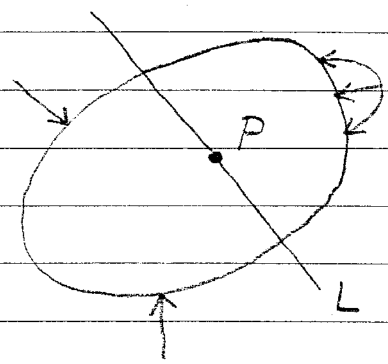
$$\sigma_c = \frac{N_c}{A_c} = \frac{2566}{890} \cong 2.88 \text{ kgf/cm}^2 \quad (T)$$

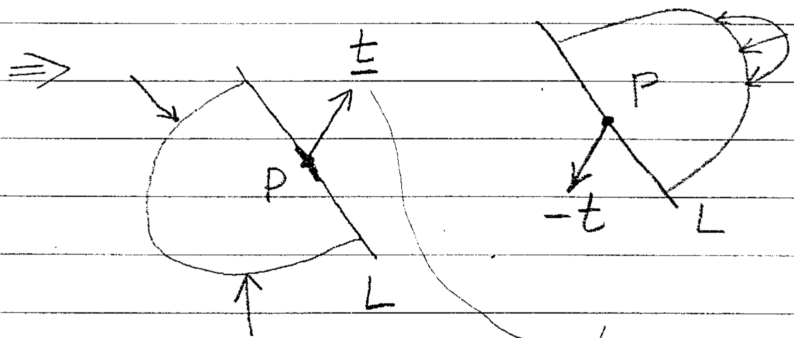
$$\sigma_s = \frac{N_s}{A_s} = -\frac{2566}{10} = -256.6 \text{ kgf/cm}^2 \quad (C)$$

CHAPTER 4

STRESS AND STRAIN ANALYSIS

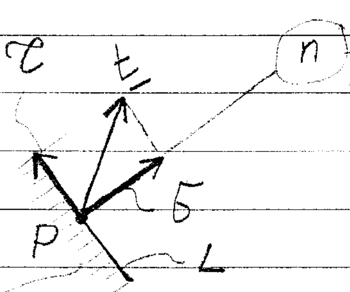
Review:





stress vector
↓
defined as the force per unit area

normal axis



$\underbrace{\sigma \rightarrow \text{normal stress}}$

the comp. of $\underline{t}$ in n dir.

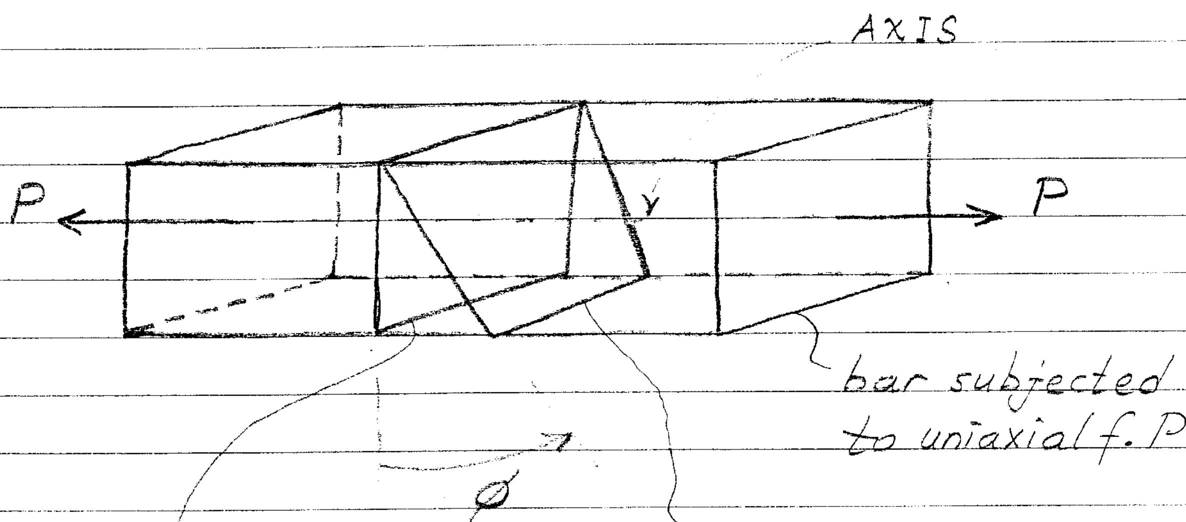
shaded part denotes the part retained after the cut

σ {
- $+ \Rightarrow$ directed outwards (T)
- $- \Rightarrow$ directed inwards (C)

$\tau \rightarrow$ $\underbrace{\text{shear stress}}_{\text{projection of } \underline{t} \text{ on } L}$ $\rightarrow$ tangential comp of $\underline{t}$

τ {
- $+$ if directed to the left as we look at in n-dir.
- $-$ if directed to the right ...

UNIAXIAL STRESS STATE

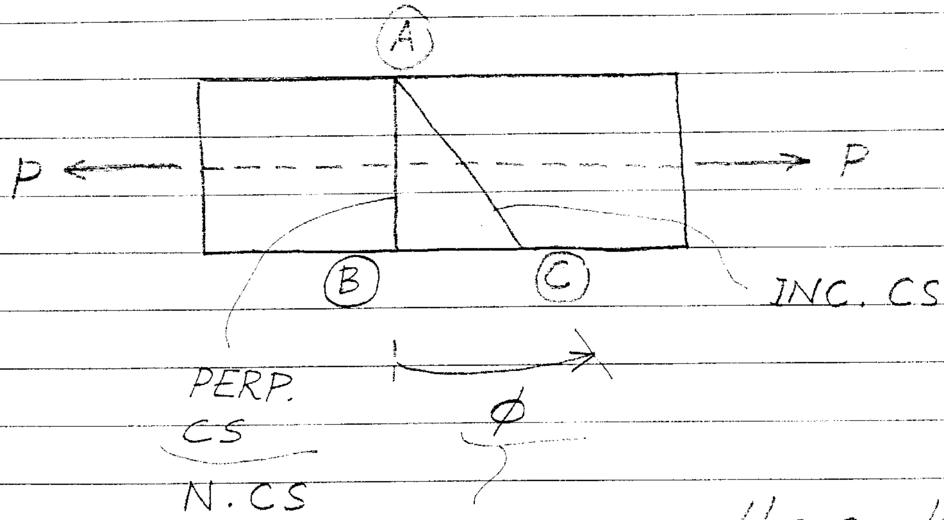


inclined CS
INC.

Perpendicular (Perp.)

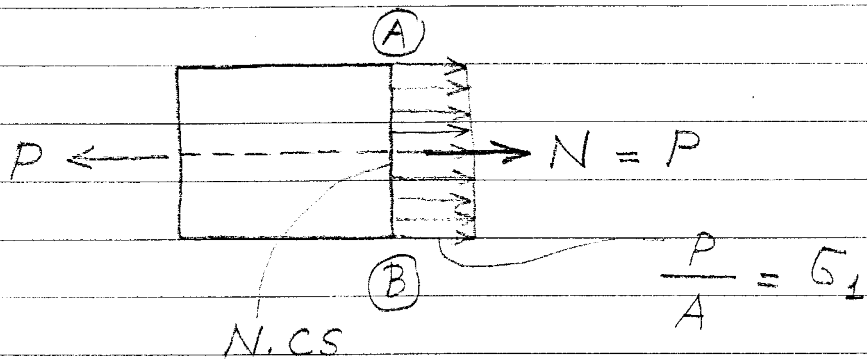
(N) - $\underbrace{\text{(normal) CS}}_{\perp \text{ axis}}$

For simplicity:



measures the angle of INC. CS from N.CS in anticlockwise dir.

Stresses on N.CS:



$A \rightarrow$ area of N. CS

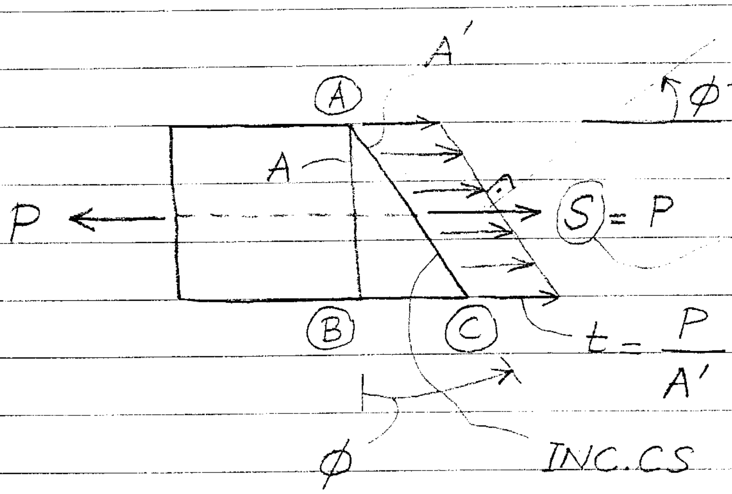
$\sigma_1 = \dfrac{P}{A} \rightarrow$ normal stress acting on N.CS

The uniaxial f. P produces only normal stress σ_1 on N.CS.

Stresses on INC. CS :

measured in anticlockwise dir.

the angle of n-axis from the bar axis directed to the right



hor. f. acting on inc. CS

$A \rightarrow$ area of N. CS
$A' \rightarrow$ area of INC. CS

$$\Rightarrow \boxed{A = A' * \cos\phi} \quad (*)$$

$S = P$ acting on inc. CS produces hor. stresses t.

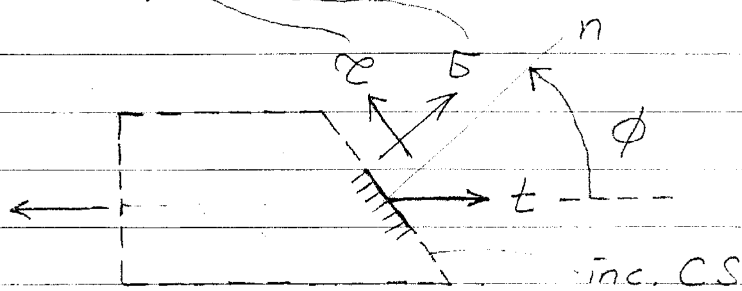
$$\Rightarrow t = \frac{P}{A'} = \underbrace{\frac{P}{A}}_{\sigma_1} \cos\phi$$

$$\Rightarrow \boxed{t = \sigma_1 * \cos\phi} \quad (**)$$

stress acting on inc. CS

hor.

(+) normal and shear stresses



Decompose t into σ and τ parts

$\Rightarrow \quad \sigma = t * \cos\phi$

$\Rightarrow \quad \boxed{\sigma = \sigma_1 \cos^2\phi}$

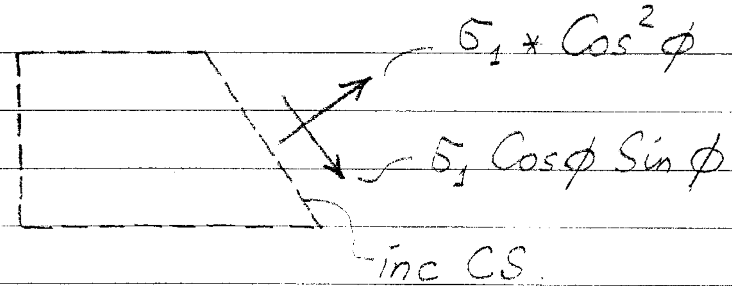
A

$\tau = -t \sin\phi$

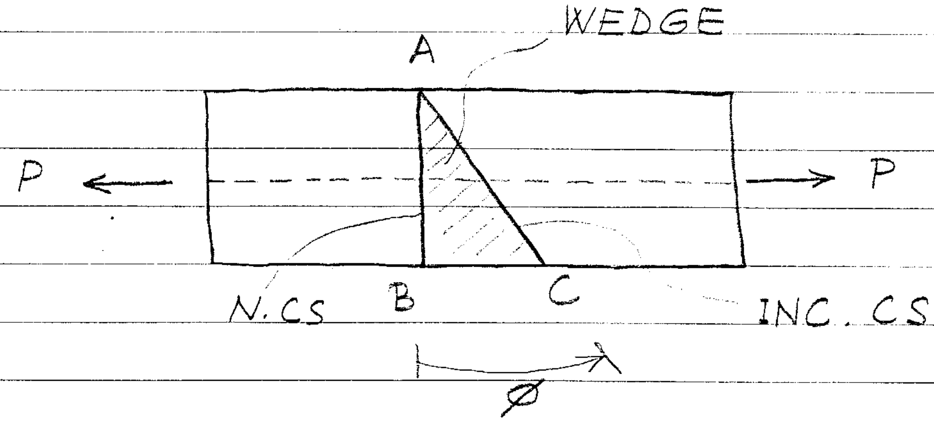
$\Rightarrow \quad \boxed{\tau = -\sigma_1 \cos\phi \sin\phi}$

formulas for σ and τ acting on inc. CS
↓
determines σ and τ in terms $\sigma_1 = \frac{P}{A}$

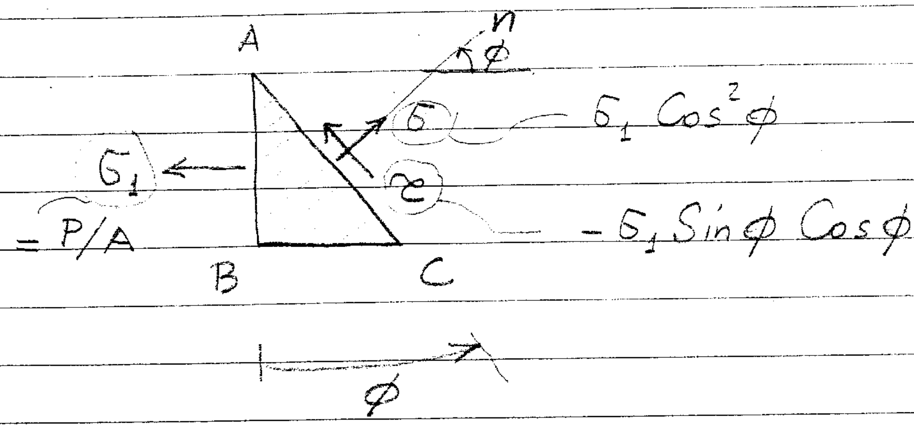
P → produces both σ and τ on inc. CS.
→ produces only the normal stress σ_1 on N. CS.



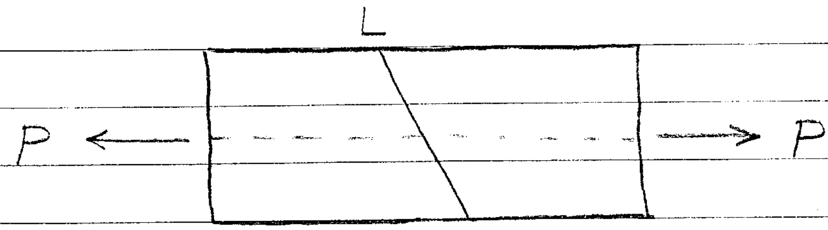
Summary:

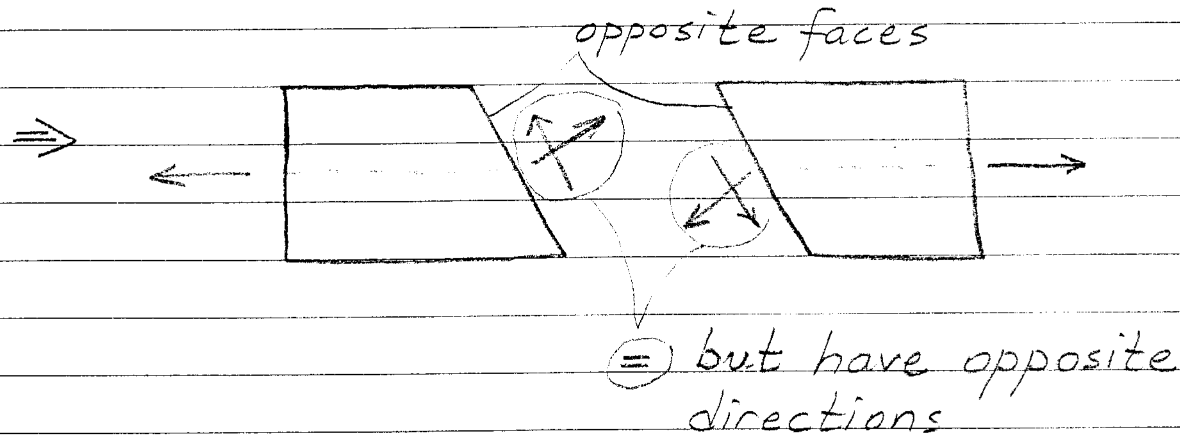


Isolate the wedge :



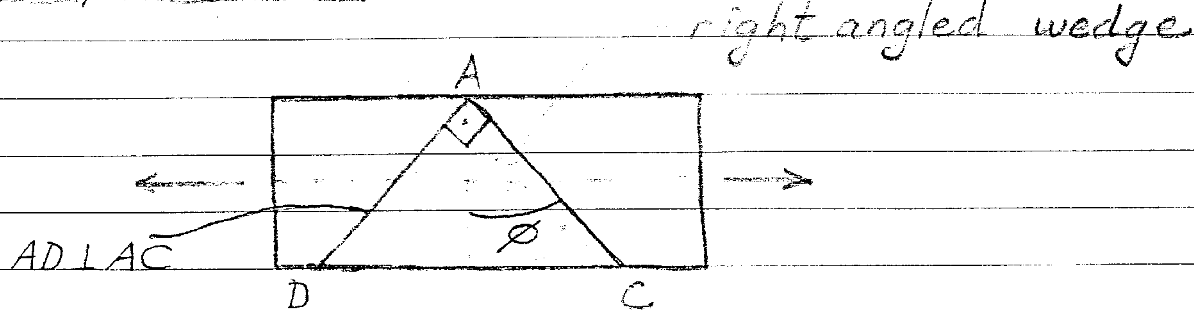
Opposite faces:



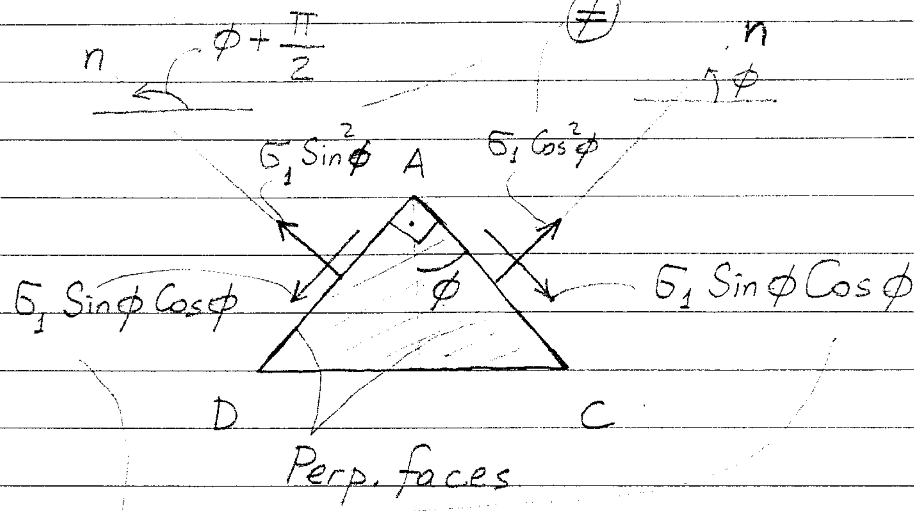


The stresses acting on opposite faces are equal, but have opposite directions.

Perpendicular faces:



Isolate the wedge:



$=$ getting away from each other

AC:

$$\phi \to \phi$$

$$(A) \Rightarrow \quad \sigma = \sigma_1 \cos^2\phi$$

$$\tau = -\sigma_1 \sin\phi \cos\phi$$

AD:

$$\phi \to \phi + \frac{\pi}{2}$$

$$(A) \Rightarrow \quad \sigma = \sigma_1 \overbrace{\cos^2\left(\phi + \frac{\pi}{2}\right)}^{\sin^2\phi}$$

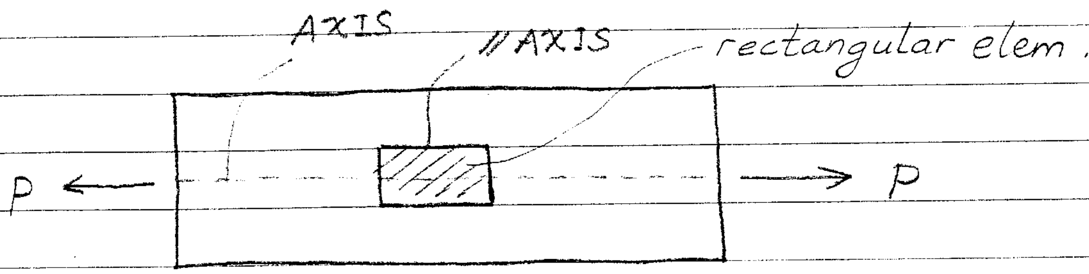
$$\tau = -\sigma_1 \underbrace{\sin\left(\phi + \frac{\pi}{2}\right)}_{\cos\phi} * \overbrace{\cos\left(\phi + \frac{\pi}{2}\right)}^{-\sin\phi}$$

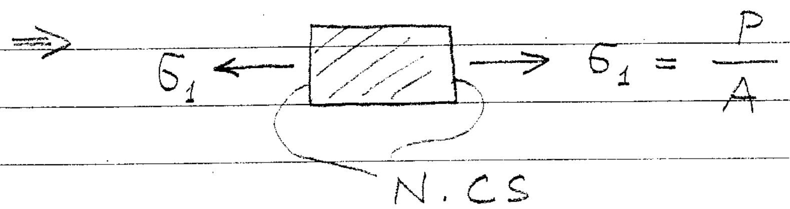
$$\Rightarrow \quad \sigma = \sigma_1 \sin^2\phi$$

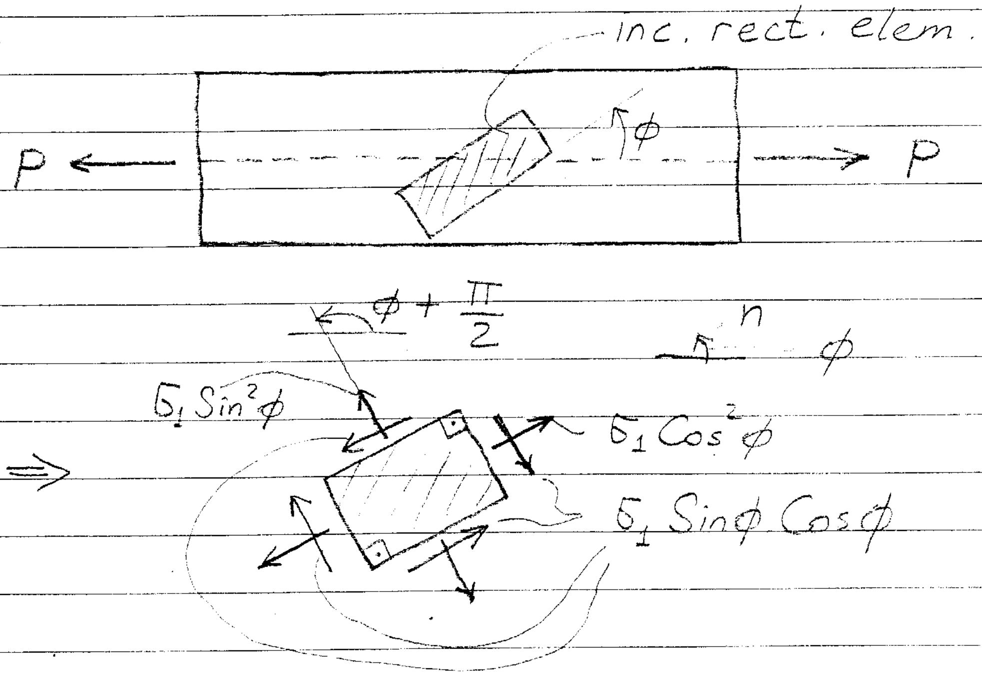
$$\tau = \sigma_1 \cos\phi \sin\phi$$

The shear stresses acting on perp. faces are equal to each other, directionwise they either approach or get away from each other.

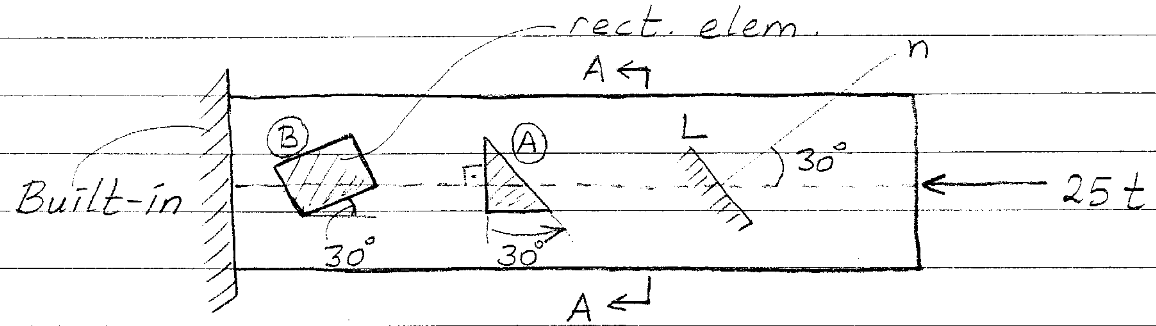
Interpretation of the rules stated for opposite and perp. faces :

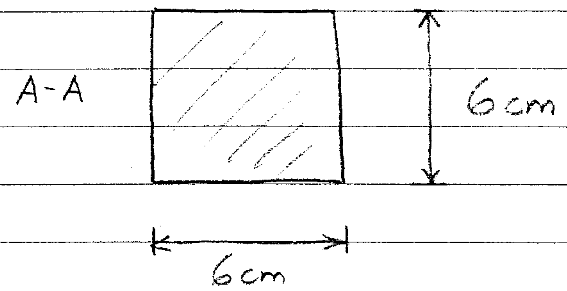






EXAMPLE :



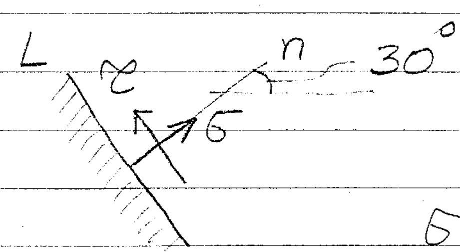


wish to find the stresses acting
a) on L
b) on the faces of elem. A
c) " " " " " B

a) compute normal stress acting on N. face :

$$\sigma_1 = \frac{N}{A} = \frac{-25000 \text{ kgf}}{36 \text{ cm}^2}$$

$$\Rightarrow \sigma_1 = -694.4 \text{ kgf/cm}^2$$



$$\sigma = \sigma_1 \cos^2\phi$$

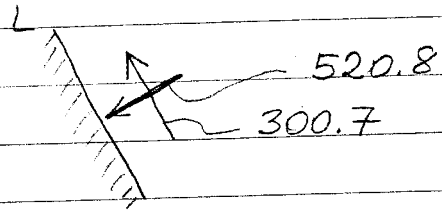
$$\tau = -\sigma_1 \sin\phi \cos\phi$$

$$\phi = 30^\circ \Rightarrow \cos\phi = \sqrt{3}/2 \ , \ \sin\phi = 1/2$$

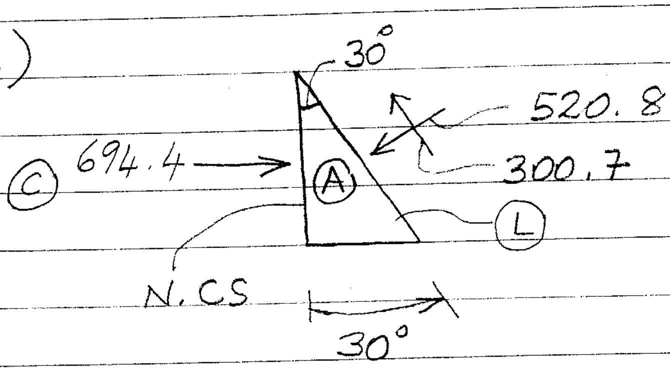
$$\Rightarrow \quad \sigma = -520.8 \ kgf/cm^2$$

$$\tau = 300.7 \ kgf/cm^2$$

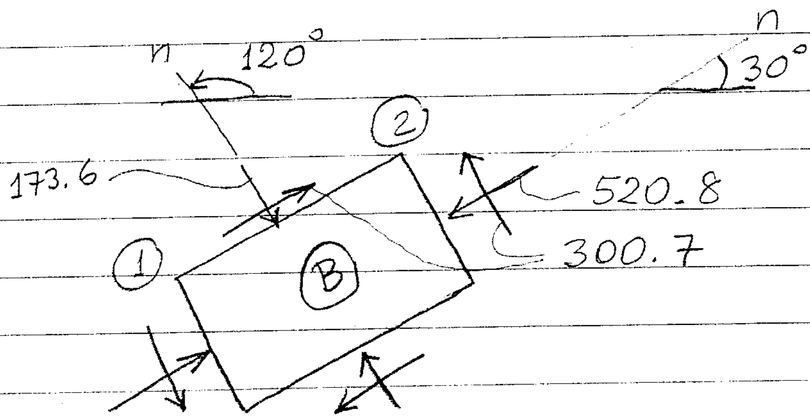
$\Rightarrow$



b)



c)

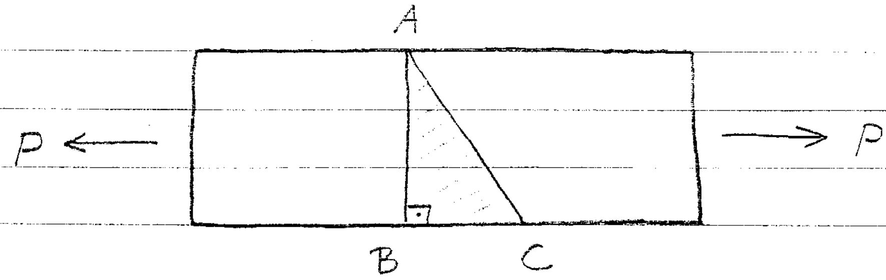


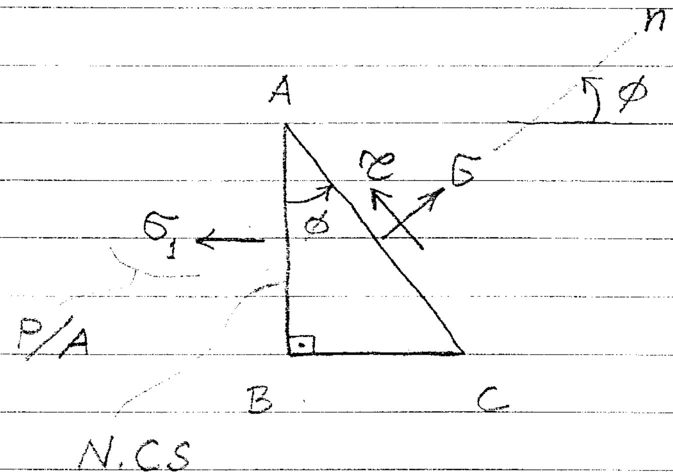
σ on the face 1 2 ?

$$\Rightarrow \phi = 120° \Rightarrow \sigma = \sigma_1 \cos^2\phi$$

$$\Rightarrow \sigma = -173.6 \ kgf/cm^2$$

MOHR CIRCLE (MC)





$$\left.\begin{aligned}\sigma &= \sigma_1 \cos^2\phi \\ \tau &= -\sigma_1 \sin\phi \cos\phi\end{aligned}\right\} (*)$$

$$\left.\begin{aligned}\cos^2\phi &= \frac{1}{2}\left(1+\cos 2\phi\right) \\ \sin\phi\cos\phi &= \frac{1}{2}\sin 2\phi\end{aligned}\right\} (1)$$

(1) $\rightarrow$ (*) :

$$\Rightarrow \left.\begin{aligned}\sigma &= \frac{\sigma_1}{2}\left(1+\cos 2\phi\right) \\ \tau &= -\frac{\sigma_1}{2}\sin 2\phi\end{aligned}\right\} (**)$$

(**) :

$$\Rightarrow \sigma - \frac{\sigma_1}{2} = \frac{\sigma_1}{2}\cos 2\phi \quad (A)$$

$$\tau = -\frac{\sigma_1}{2}\sin 2\phi \quad (B)$$

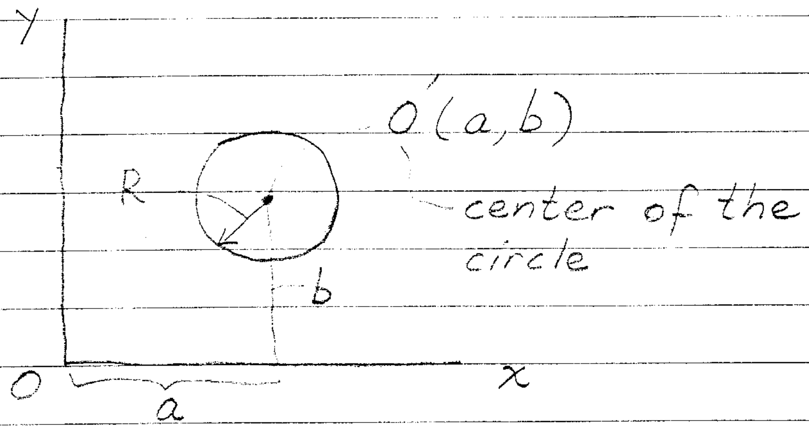
$(A)^2 + (B)^2$:

$$\Rightarrow \left(\sigma - \frac{\sigma_1}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_1}{2}\right)^2 \underbrace{\left(\cos^2 2\phi + \sin^2 2\phi\right)}_{=1}$$

$$\Rightarrow \boxed{\left(\sigma - \frac{\sigma_1}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_1}{2}\right)^2} \quad (A)$$

Recall eq. of a circle in xy plane:

$$(x-a)^2 + (y-b)^2 = R^2$$



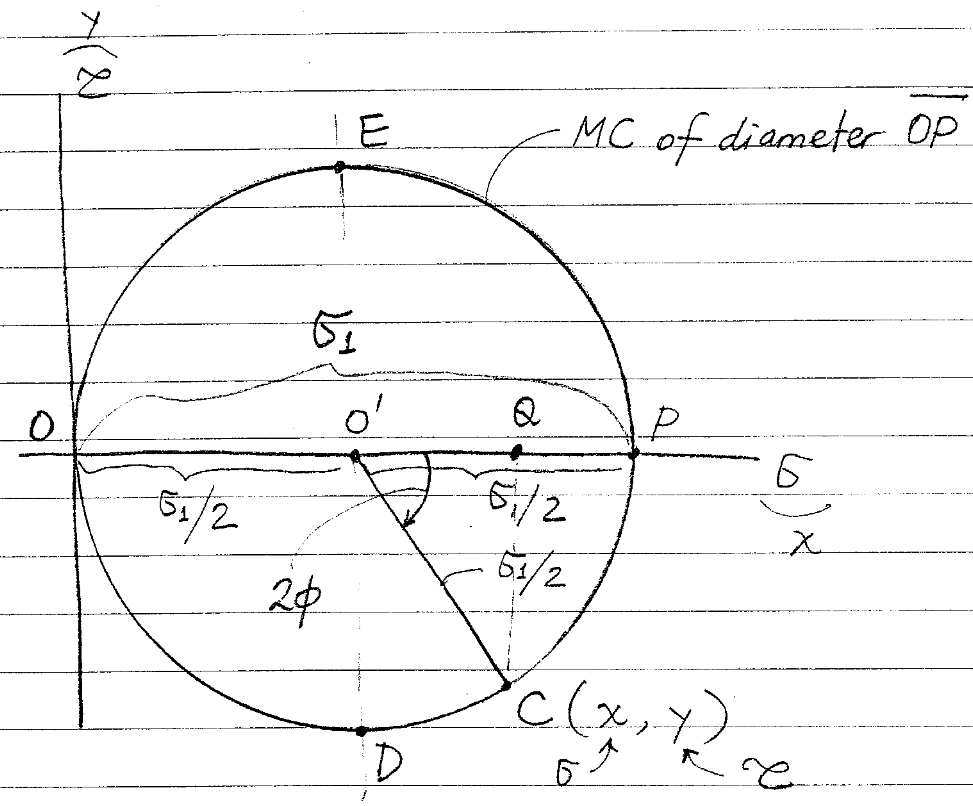
In (A) $\rightarrow$ $\sigma \rightarrow x$

$\tau \rightarrow y$

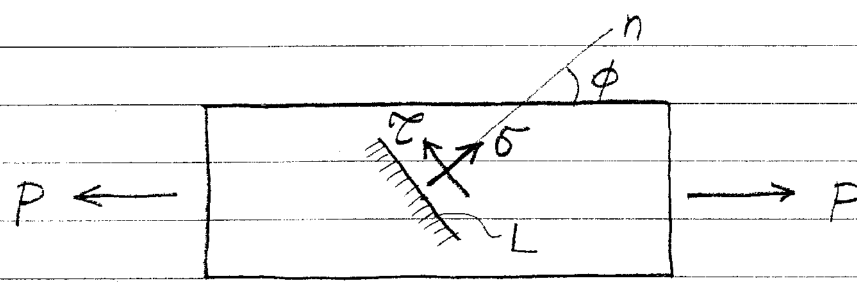
$\Rightarrow$ (A) describes a circle in $\sigma - \tau$ plane

$$\Rightarrow \quad R \rightarrow \frac{\sigma_1}{2}$$

$$\underbrace{\text{center}}_{O'} \rightarrow \left(\overset{a}{\frac{\sigma_1}{2}} , \overset{b}{0} \right)$$



To find σ & τ acting on L by using MC, we follow the steps:



1) find $\sigma_1 = \dfrac{P}{A}$

2) construct MC

3) take the angle $2*\phi$ from the line $\overline{O'P}$ in the clockwise direction which determines the point C on MC. The coordinates of the point C give the stresses σ and τ.

$$C \rightarrow (\overbrace{x}^{\sigma}, \underbrace{y}_{\tau}) \qquad (C)$$

Proof of (C):

$$x = \underbrace{\overline{OO'}} + \underbrace{\overline{O'Q}} \qquad (**)$$

$$= \frac{\sigma_1}{2} + \frac{\sigma_1}{2} \cos 2\phi \overset{(**)}{=} \sigma$$

$$y = -\overline{QC} = -\frac{\sigma_1}{2} \sin 2\phi \underset{(**)}{=} \tau$$

$\Rightarrow$ proof is completed.

ABSOLUTE MAX SHEAR STRESS (SS)

$$\text{Absolute max } SS = \underbrace{\max|\tau|}_{\tau_{max} \rightarrow ?}$$

Each L corresponds to a pt. on MC.

As L changes $\rightarrow$ ϕ will change, the location of C will change.

τ will reach its max abs. value at D and E

$$\Rightarrow \boxed{\tau_{max} = \frac{|\sigma_1|}{2}}$$

Study D & E separately:

D:

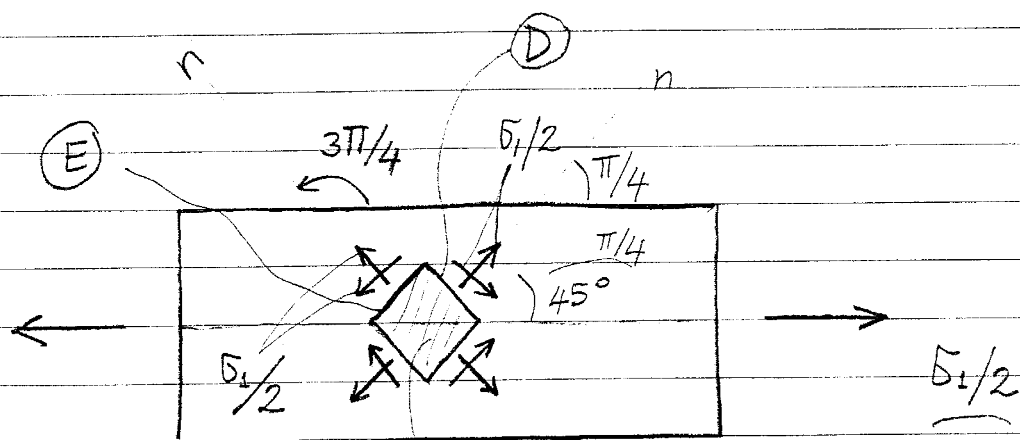
$$\Rightarrow 2\phi = \frac{\pi}{2} \Rightarrow \phi = \frac{\pi}{4}$$

$$\Rightarrow \sigma = \frac{\sigma_1}{2}, \quad \tau = -\frac{\sigma_1}{2}$$

E:

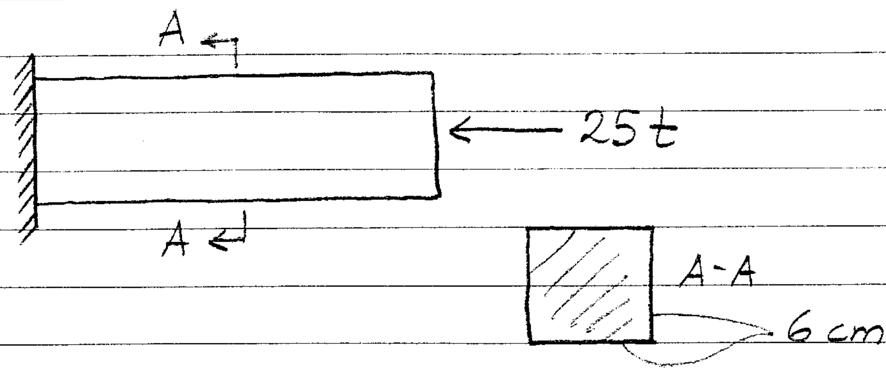
$$\Rightarrow 2\phi = \frac{3\pi}{2} \Rightarrow \phi = \frac{3\pi}{4}$$

$$\Rightarrow \sigma = \frac{\sigma_1}{2}, \quad \tau = +\frac{\sigma_1}{2}$$



The element in which τ_{max} acts.

EXAMPLE: (considered previously)



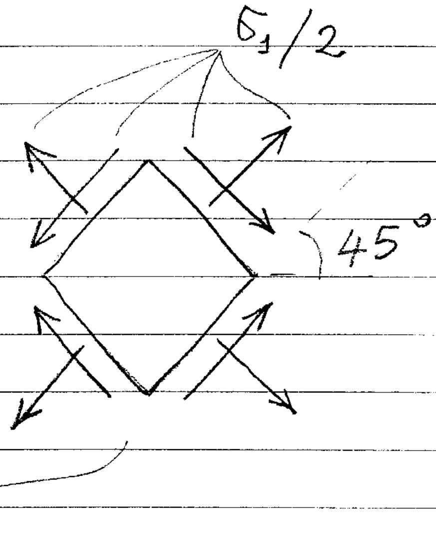
wish to find τ_{max} and the elem. in which it is acting.

$$\sigma_1 = \frac{N}{A} = \frac{-25\,000 \text{ kgf}}{36 \text{ cm}^2} = -694.4 \text{ kgf}/\text{cm}^2$$

$$\Rightarrow \frac{\sigma_1}{2} = -347.2 \text{ kgf}/\text{cm}^2$$

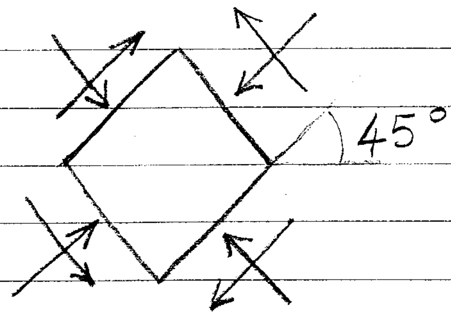
$$\Rightarrow \tau_{max} = \frac{|\sigma_1|}{2} = 347.2 \text{ kgf}/\text{cm}^2$$

The element:



These directions are valid when $\sigma_1 > 0$

in our problem:



PLANE STATE OF STRESS

xy plane coincides with the midplane of the plate

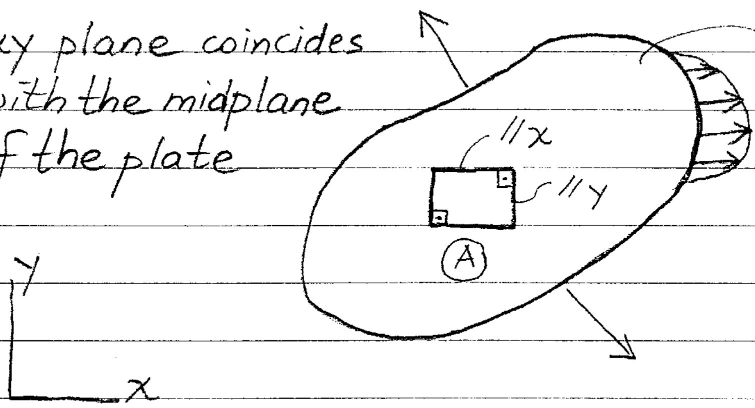
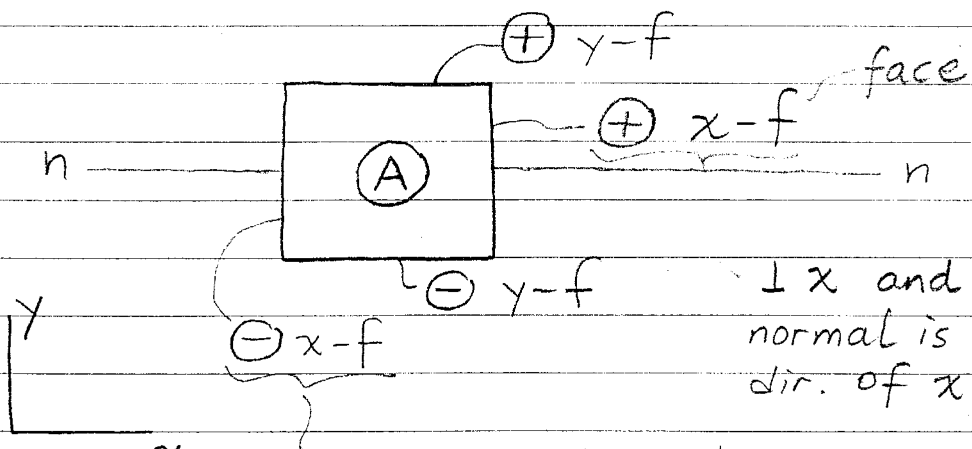


plate subjected to forces on its plane

Isolate rectangular element:



⊥ x and its outer normal is in the ⊕ dir. of x

⊥ x and its outer normal is in the ⊖ dir. of x